THE

HISTORY

OF

GUNNERY,

With a NEW METHOD of deriving the THEORY of PROJECTILES in VACUO, from the properties of the SQUARE and RHOMBUS.

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TO THE

RIGHT HONOURABLE

LORD VISCOUNT TOWNSHEND,

Maſter General of the Ordnance, &c. &c.

This Performance is, with the greateſt reſpect and eſteem,

dedicated by

His moſt obedient,

humble Servant,

JAMES GLENIE.

PREFACE.

THE author's deſign in this performance, is not only to lay before the reader an hiſtorical account of the different diſcoveries which have been made relative to the reſiſtance of the air, by the moſt eminent writers who have treated of this ſubject; but likewiſe, to give the theory of projectiles *in vacuo*, derived in a new manner from very ſimple principles, with a method of reducing projections on inclined planes, to thoſe which are made on the horizon. He is perſuaded, that, were the law of the air's reſiſtance once exactly aſcertained, which is undoubtedly capable of an accurate determination, a competent ſhare of knowledge in mechanics would enable ſkilful Engineers to ſurmount all other difficulties in directing the management of artillery. The reſiſtance of mediums, it is true, is one of the moſt difficult ſubjects to which mathematics have ever been applied. The difficulties attending it have been ac-

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knowledged by all who have conſidered it, and were ſenſibly felt by Sir Iſaac Newton himſelf. He has certainly diſcovered great ingenuity and philoſophical invention on this ſubject. But his concluſions are only applicable to very ſlow motions, and are not delivered by him in a form fitted for practice. Gunnery, in its preſent ſtate, is only a ſort of random or gueſs-work; and ſuch it muſt continue to be, till the theory of the air's reſiſtance be accurately inveſtigated.

The author once intended to have taken this opportunity of publiſhing ſeveral calculations relating to the reſiſtance of mediums in general, and ſome to that of the air in particular, between which, and certain experiments made with great accuracy, there was a ſurpriſing degree of coincidence. But he thought it would be better to defer the publication of theſe computations, till an opportunity ſhould preſent itſelf of trying and examining them by a greater variety of experiments. However, the reader will eaſily

underſtand,

underſtand, from the problems ſubjoined to the hiſtorical account, that this is a ſubject on which the author has beſtowed a good deal of attention. And, although he has endeavoured to give the account itſelf in as plain a manner as poſſible, in order to render it univerſally intelligible; yet, mathematical readers will find ſufficient ſcope for the exerciſe of all their ſkill, in attempting the ſolutions of theſe problems. Several of them, indeed, it muſt be acknowledged, do not relate immediately, either to the air, or to any medium perhaps exiſting in nature. But he thought it would not be altogether improper to inſert them, ſince their ſolutions may be very much facilitated by the general problems which he ſent ſome time ago in a paper to the Royal Society, with an intention to extend indefinitely geometrical compariſons, which are uſually confined to three gradations, by geometers both ancient and modern.

In

In the demonſtrative part, which relates to the theory of projectiles *in vacuo*, the young mathematician will find a method put in practice, of firſt conſidering the properties of ſimple rectilinear figures, and of afterwards applying them to curves which can be generated by the motion of the ſides or angles of ſuch figures. Simplicity has been the principal thing which the author has all along aimed at in this performance. The mathematical reader, then, muſt only conſider it as a prelude or forerunner to ſomething which is to follow, relating to the reſiſtance of mediums in general, the moſt difficult part of mixed mathematics, and to that of the air in particular, by the aſcertaining of which alone Gunnery can be rendered perfect and complete.

ERRATA.

P. 83. *l.* 3. *for* Coles *read* Cotes.
P. 88. *for* TM *read* TH.
P. 102. *l.* 13. *for* rhombuſes *read* rhomboids.
P. 104. *l.* 5. *for* angle *read* angles.
P. 118. *l.* 22. *for* OK *read* O, K.
P. 142. *l.* 18. *for* OR *read* OZ.

THE

HISTORY

OF

GUNNERY.

PART I.

FOR ſome time after men began to apply gun-powder to military purpoſes, their machines and pieces of ordnance were very ponderous and unwieldy, and of courſe altogether unfit for quick or expeditious ſervice. Military people, at that time, poſſeſſed but a ſmall ſhare of learning of any kind, and almoſt none at all of a mechanical or mathematical nature. What they did in their profeſſion, in relation to the management of artillery, was entirely the effect of practice, and a bare repetition of what

they ſaw every day done. The form of their artillery, as well as of the warlike engines and inſtruments which they made uſe of in conducting it, was only ſuch as the moſt obvious incidents ſuggeſted, or the rudeſt and moſt uncultivated invention dictated. Their firſt pieces were very cumbrous, being of a clumſy, and almoſt unmanageable make; for, as they ſucceeded to the machines of the antients, they were employed, like them, in throwing ſtones of a prodigious weight, and therefore were neceſſarily of a huge and enormous bore, conſiſting uſually of pieces of iron fitted together length-ways, and hooped with iron rings. Some of them were ſo large, that they could not be fired above four or five times a day. Such were thoſe with which, we are told, Mahomet II. battered the walls of Conſtantinople, in the year one thouſand four hundred and fifty-three, being ſome of them of the calibre of no leſs than twelve hundred pounds; and Guicciardin, in the firſt book of his hiſtory, informs us, that ſo large a portion of time intervened between

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the different chargings and dischargings of one of those pieces, that the besieged had sufficient time to repair, at their leisure, the breaches made in their walls by the shock of such enormous stones.

But as mathematical knowledge increased in Europe, that of mechanics gradually advanced, and enabled artists, by making brass cannon of a much smaller bore for receiving iron bullets, and a much greater charge of strong powder, in proportion to their calibres, to produce a very material and important change in the construction and fabric of those original pieces. Accordingly, this historian, in the same book of his history, informs us, that, about a hundred and fourteen years after the first use made of those unwieldy pieces by the Venetians, in the war which they carried on against the Genoese, in the year one thousand three hundred and eighty, the French were able to procure, for the invasion of Italy, a great number of brass cannon, mounted on carriages, drawn by horses; and that these

pieces could always keep pace with the army.

In enumerating the advantages which arose from this alteration, he obſerves, that they were pointed with incredible quickneſs and expedition in compariſon of thoſe formerly made uſe of in Italy, were fired at very ſmall intervals of time, and could produce, in a few hours, an effect which thoſe others could not have produced in the ſpace of many days. His words are, ‘ Condotte ‘ alle muraglie erano piantate con preſtezza ‘ incredibile, et interpondaſi dall' un colpo ‘ all' altro piccoliſſimo intervallo di tempo, ‘ ſi ſpeſſo, et con impeto ſi gagliardo per‘ cuotevano, che quello che prima in Italia ‘ fare in molti giorni ſi ſoleva, da loro in ‘ pocchiſſime hore ſi faceva :' And mathematical ſtudies muſt have made conſiderable progreſs on the continent by that time, ſince Tartalea, the inventor of the method of ſolving cubic equations, which is uſually aſcribed to Cardan, and which is the only general method we have of ſolving them at this

this day, about forty-three years after this change took place, commenced author at Venice.

This change in the formation of artillery has as yet undergone no very material alteration. Lighter pieces, indeed, are now employed than thoſe which were made uſe of at firſt; but they have ſuffered greater variations in reſpect of ſize than proportion.

The reſiſtence of the air renders all concluſions drawn from experiments made with any of the pieces of cannon now in common uſe, very uncertain and precarious; for this medium, beſides retarding the progreſſive motion of a bullet, gives it a lateral deflexion or deviation, when fired from one of theſe pieces, which makes it meet the ſurface of the earth at a conſiderable diſtance often from the point, where a plane, paſſing through the line of projection, perpendicular to the horizon, meets it, and on either ſide too of that plane.

This

This lateral deflexion, I know from experience, is effectually remedied at ſmall elevations, by the uſe of pieces properly rifled, and of balls with knobs on them to fill exactly the grooves or rifles. And a very ſmall ſhare of ſagacity will eaſily enable any one to prevent this inconvenience, at all elevations, by varying a little the form of the bullet.

And as this is a matter of ſo great importance, to have every part of the path of the bullet in the plane paſſing thro' the centre of the earth and the line of projection, and may be ſo eaſily obtained, it is well worth the attention of engineers and artilleriſts. For, when this deflective force of the air is corrected, and, of conſequence, all rotatory motion in the direction of the bullet is deſtroyed, they have little more to do, as far as their profeſſion is connected with the management of artillery, than to determine the law of its reſiſtence and the firſt velocity of the bullets, and to adapt their calculations relating

relating to theſe matters to ready and expeditious practice.

And though theſe two problems have never yet been perfectly and accurately ſolved, or at leaſt publiſhed to the world with complete ſolutions, they ought not to deſpair of being able to ſolve them, but ſhould rather be encouraged and animated by what others have done towards the ſolution of theſe queſtions, to do more, and ſhould ſhow that they are at leaſt willing to contribute their ſhare for the completing of that art which they profeſs, and the perfecting of its rules and maxims. The undertaking is indeed difficult; but I know that ſucceſs in it is not unattainable. Every engineer whatever ought to have a very conſiderable ſhare of mathematical knowledge, to enable him in ſome meaſure to enter into ſuch reſearches, and to conduct him in his experimental inquiries.

As to the reſiſtance of mediums, when men firſt began to think of this ſubject, I cannot

cannot pretend to determine; but it is certain, that the consideration of it took place as far back at least as the time of Aristotle, who presided over Europe for so many ages with almost unlimited sway. His positions, however, with regard to this matter are made more with an intention to refute the opinions of some of the antients, who held a vacuum to be absolutely necessary to motion, than with any real design to investigate the quantity, nature, or laws of such resistance, or to determine the degrees of retardation which a body would meet with in moving through them. He formed conjectures, without either confirming or examining them by experiment. And it is indisputably certain, that conjecture, without experiment to try and sift it, is as apt to lead one into error, as random-experiment, without sagacious conjecture to regulate and conduct it, bids fair for producing nothing of the least importance.

But, if it must be confessed that many of the antient philosophers gave rashly into the first

first of these extremes, it must likewise be acknowledged, that there are many now, who give as freely into the last, and, prostituting the powers which God has bestowed upon them, spend their lives in making experiments, in a great measure, at random, without having one useful principle or idea in their minds, which they wish to examine or establish by them. These persons are generally such as are so little acquainted with the science of quantity and proportion, as to be altogether unable to apply metricks to any thing which occurs, or to know, from calculation beforehand, what ought to be the result of their experiments, on the supposition that the hypothesis or conjecture which they intend to try by them, is true.

With such people, their understandings in experimental cases must keep pace with their hands, not for want of abilities, but from want of reflection, and a misapplication of them. They intirely misunderstand the plan of experimenting hinted at by Lord

Verulam, and ſo ſucceſsfully carried into execution by Sir Iſaac Newton, and ſeveral other ingenious men, whoſe ſagacious conjectures generally preceded, and gave riſe to their particular experiments, and who were always able, when any thing new accidentally occurred, to examine it by a ſort of certain and mathematical meaſurement, and puſh it into concluſions, by means of computations founded on the moſt unerring principles.

Now, Ariſtotle delivers the ſubſtance of his theory of reſiſtance, which was erected to deſtroy the opinion of the neceſſary exiſtence of a vacuum for motion, in two propoſitions, one of which relates to the motions of bodies of different gravities, or weights, in the ſame medium; and the other to the motion of the ſame body in different mediums. As to the firſt, he aſſerts, that bodies of different weights or gravities, move in the ſame medium with unequal velocities, which have to each other

reſpectively

respectively the ratios of their gravities or weights.

That ſuch an abſurd propoſition ſhould have kept its ground ſo long, is aſtoniſhing. For, according to him, an iron ball of an hundred pounds weight, deſcending through an height of fifty yards, ought to reach the ground before another of a pound weight has deſcended above one yard. But we find from experience that they reach it nearly at the ſame inſtant of time.

Concerning the ſecond, he affirms that the ſame body moves in different mediums, with velocities which have to each other reſpectively the reciprocal ratios or proportions of the denſities of thoſe mediums. And the concluſion he draws from it is this, that, ſince a vacuum differs infinitely from any, even the rareſt or moſt ſubtile medium, if a body in any medium run through a certain ſpace in a certain portion of time, it ought to run through the ſame ſpace in a vacuum in an inſtant. But inſtantaneous motion

through

through a ſpace is impoſſible. Therefore it is likewiſe impoſſible to have recourſe to a vacuum on account of motion.

But this argument, even allowing his premiſſes to be true, does not ſtrike at the exiſtence of a vacuum or empty ſpace, but only at the exiſtence of motion in it. Both theſe propoſitions, however, Galileo, in his firſt dialogue, has ſhown to be falſe and contrary to experiment; and this too he has done by experiments made in different mediums. And, after giving an ample and ſatisfactory confutation of Ariſtotle's notions with regard to reſiſtance, he goes on with the proſecution of a very pretty thought or idea, which he produces ſeveral examples to eſtabliſh from the different velocities of bodies of different ſpecific gravities, in reſiſting mediums of different denſities. It is this, that all bodies would deſcend *in vacuo* with equal degrees of velocity, and fall through equal ſpaces in the ſame time.

Obſerving

Obſerving that the difference of velocities in moveables of different ſpecific gravities was always the greater, the denſer the medium was in which they deſcended, and that, as the medium became rarer in which the experiment was tried, this difference became leſs, he very naturally, though ſagaciouſly, concluded, that, were the medium indefinitely rare or ſubtile, this difference would be indefinitely ſmall, and that, *in vacuo*, it would intirely vaniſh, and leave the velocities in a perfect ſtate of equality. This was a very pretty and natural induction; and, though it may ſeem now very eaſy and ſimple when made, it muſt have required no ſmall ſhare of ſagacity at firſt to have made it. He found, that all moveables, both light and heavy, under the ſame form, deſcended from very ſmall diſtances above the earth's ſurface, with nearly the ſame velocity, and through the ſame ſpace, nearly in the ſame time; and very judiciouſly obſerved, that the great differences which take place between the diſtances they deſcend through, when the times of deſcent

are great, could not arise from their specific gravities or weights under the same bulk, but from the medium itself, since the lines of descent, in that case, would always observe the same proportion, which we find neither happens to bodies of the same, nor of different specific gravities.

This discovery of the equal descent of all moveables *in vacuo*, is of the utmost importance in natural philosophy, and frees the mind from the consideration of a circumstance which would serve to embarrass and perplex her in contemplating the nature of resistance in mediums. For, once being certain that all bodies would move in the same manner *in vacuo*, where there is no medium, we are sure that all those varieties, which are observable in the motions of different bodies, in the same or different mediums, must arise from these mediums themselves.

He likewise inferred, from the nature of resistance, by a very simple train of thought, that

that bodies deſcending in any uniform medium, by a motion which would be uniformly accelerated from a ſtate of reſt, were they to deſcend *in vacuo*, would at length, when the reſiſtance exactly equalled the accelerating force, acquire velocities which this force could not poſſibly increaſe, and with which theſe bodies would continue to deſcend in that medium for ever. And this would happen to bodies deſcending from reſt in the air, were it of the ſame denſity at different diſtances from the earth, and if the force of gravity were uniform, as this juſtly celebrated philoſopher ſeems to have imagined; which indeed is not at all ſurpriſing, ſince its uniformity appears in all the experiments we make in the ſame place, at heights not far from the ſurface of the earth. But, ſince neither of theſe ſuppoſitions is true, the caſe will be in ſome meaſure altered in relation to the deſcent of bodies in our atmoſphere.

Galileo likewiſe diſcovered the reaſon why ſmall bodies meet with greater degrees of retardation

retardation in any medium than larger ones of the same densities and figures; by considering that the solidities of bodies are in the sesquialteral proportion of their surfaces, and, consequently, in large bodies, bear a greater proportion to their surfaces than in small ones. But, though he has sufficiently evinced the falsity of Aristotle's two propositions, with regard to the resistance of mediums, he does not seem to have reached the truth in those, which he has substituted in their place.

He proposes a method for determining the proportion of the spaces, through which bodies of different specific gravities will descend in any medium in the same time, by a subtraction or diminution of weight; which method may be comprehended in the following rule: Let there be a fluid, and any number of bodies of the same bulk and figure, specifically heavier than this fluid; divide the numerical value of the specific gravity of the heaviest of these bodies, by those of the rest respectively, calling that of the fluid

unity,

unity, and ſubtract the quotients ſeverally from the ſpecific gravity of the heavieſt; then the remainders ſhall reſpectively have to the difference of the ſaid ſpecific gravity and that of the fluid, the ratios of the ſpaces which theſe other bodies ſeparately deſcend through, to that which the heavieſt deſcends through, in the ſame time.

Here, however, it muſt be acknowledged, that this curious inquirer into nature is conſiderably miſtaken; and he is not more ſucceſsful in his method of aſcertaining the velocities of the ſame body in different mediums, and the ſpaces it would deſcend through in them ſeparately in equal times, which is ſubſtituted inſtead of Ariſtotle's ſecond propoſition. However, he does not propoſe them as abſolutely true and certain, but delivers them under the perſuaſion, that they will come nearer to the truth than the propoſitions advanced by Ariſtotle. He does not ſeem to have had a diſtinct idea of the quantity of reſiſtance in fluids, or to have imagined it to be capable of accurate meaſurement,

ſurement, and reducible to certain laws. For, in his fourth dialogue, after taking notice of the ſmall inequalities, that would ariſe from gravity's not acting in parallel lines, on a projectile in different points of the curve, which it deſcribes in its flight, and from the different diſtances of the ſeveral parts of the horizontal plane, from the center of the earth; when he comes to ſpeak of the irregularities ariſing from the reſiſtance of the air, he expreſſes himſelf in the following manner:—'As to the irregularity 'proceeding from the impediment of the 'medium, this, we grant, is more conſi'derable, and, by reaſon of its ſo manifold 'varieties, cannot poſſibly be reduced to 'certain laws.' And afterwards he ſays, 'Whereas theſe accidents of gravities and 'figures are ſubject to infinite mutations, 'we can come at no certain knowledge con'cerning them.' Beſides, he appears to have conceived the reſiſtance of the air to be much leſs than it really is. For, though he ſeems to have been ſenſible enough, that a body moving ſwiftly in it, would be more re-

retarded, than another of the ſame bulk, form, and denſity, moving but ſlowly, he expreſsly concludes, from ſome experiments which he made on pendulous bodies, with ſmall velocities, that this difference of retardation is not great. He likewiſe affirms, that a heavy body, of a ſpherical form, or any body of a cylindric form, will nearly trace out in the air a parabola. Take his own words. 'But, among thoſe projects, 'which we make uſe of, if they are of 'a round form, nay, if they are of a 'lighter matter, and have a cylindric form, 'ſuch as are arrows thrown from bows, 'their track or path will not ſenſibly deviate from the curve of a parabola.'

After Galileo had thus concluded, that no conſiderable variation could ariſe from the reſiſtance of the air, in the flight of heavy bullets, ſuch as ſhells and cannon-ſhot, his concluſion was univerſally acquieſced in, as well by mathematicians, as by artilleriſts, writers on gunnery and practitioners in it ; till Sir Iſaac Newton conſidered the doctrine of reſiſtance in fluids, after a more ſubtile, ſcientific, and mathematical method,

thod, than any who went before him had done. In the ſecond book of his *Philoſophiae naturalis principia mathematica*, which forms by far the moſt abſtruſe and difficult part of that immortal work, he treats profeſſedly of the reſiſtance of fluids. It plainly appears from his demonſtrations, that the reſiſtance of the air is much greater to bodies moving in it, than what Galileo and thoſe who followed him imagined; and that it ought by no means to be neglected in gunnery.

He conſiders three different caſes of reſiſtance in a medium which is uniform or ſimilar throughout. The firſt of theſe is, when it reſiſts in the ratio of the velocity; the ſecond, when it reſiſts in the duplicate ratio of the velocity; and the third, when it reſiſts partly in the ratio of the velocity, and partly in the duplicate ratio of the ſame; and theſe, in the order I have now mentioned them, he treats of reſpectively in the firſt, ſecond, and third ſections of that book.

In the ſecond propoſition of the firſt ſection,

tion, he examines the firſt of theſe reſiſtances, abſtracting from gravity or any other force, which might divert the moving body from its original direction. In the third, he conſiders the motion of a body in ſuch a medium aſcending or deſcending perpendicularly to the horizon, and acted on by an uniform gravity. And, in the fourth propoſition, which is the laſt of that ſection, he conſiders the effect that would be produced in the ſame fluid, by uniting or combining both theſe motions together.

It is obvious from his demonſtrations of theſe propoſitions, that, even allowing the reſiſtance of the air to be no greater than that which is proportional to the velocity of the moveable, we muſt confeſs, that military projectiles, which fly with an incredible degree of ſwiftneſs, in compariſon of thoſe bodies which we daily diſcern performing their motions around us, would trace out paths in it, deviating very conſiderably from thoſe parabolic curves, which they would deſcribe *in vacuo*. Our author, however, in the ſcholium at the end of this ſection,

tion, obſerves, that this ſuppoſition is more a mathematical than a natural hypotheſis; and that the poſition of a reſiſtance in the duplicate ratio of the velocity, or proportional to the ſquare of it, comes nearer to the truth, and approaches much more nearly to that which actually takes place in nature.

This ſort of reſiſtance, then, he proceeds to examine in the ſecond ſection; in the three firſt propoſitions of which, he inquires into the motion of a body in an uniform or ſimilar medium, reſiſting according to this law, whilſt it is not urged by gravity or any other force, out of the line of its firſt direction, and determines ſome things, relating particularly to ſpherical bodies, moving in ſuch a fluid. In the fourth, he inveſtigates the motion of a body in ſuch an uniform medium, whilſt it aſcends or deſcends perpendicularly to the plane of the horizon, and is influenced by an uniform gravity. And, in the fifth, he determines the proportion between the times of aſcent and deſcent, by means of ſectors of a circle and hyperbola. The path, however, which a body would de-

ſcribe

ſcribe under the influence of theſe two motions, Sir Iſaac has not determined. But, in the ſixth propoſition of this ſection, he has ſhown how, in a medium, reſiſting as its denſity and the ſquare of the velocity conjointly, to determine in each point its denſity, when a body can move through it in any given curve, and likewiſe its reſiſtance and the velocity of the body, in every point of the ſame curve.

After conſidering the motion of a body in a ſimilar medium, reſiſting in the duplicate ratio of the velocity, either when it proceeds in a rectilinear direction, or when it aſcends or deſcends perpendicularly to the plane of the horizon, and is acted on by an uniform gravity; he enters, in the third ſection, into the conſideration of the ſame caſes of motion in a medium which reſiſts partly in the ratio of the velocity, and partly in its duplicate ratio. The firſt of theſe caſes, he examines in the firſt and ſecond propoſitions of that ſection; and the ſecond, in the third and fourth. And, after inveſtigating, in the fourth, fifth, and ſixth ſections reſpectively,

tively, the circular motion of bodies in resisting mediums; the density and compression of fluids; and the motion and resistance of funependulous bodies, he comes, in the seventh, to treat of the motion of fluids, and the resistance of projectiles.

This wonderful man, who, on many occasions, discovers an uncommon share of sagacity in conducting his inquiries into nature, by beginning with the most simple cases and suppositions, entirely separate and disengaged from all those minute and collateral circumstances, which might disturb his computations, and proceeding gradually from them to the examination of those cases, which are more complex and diversified, and approach nearer to real and natural occurrences, has here begun with the consideration of a discontinued fluid, consisting of particles equal among themselves, and disposed at equal distances from each other.

Since, in such a fluid, the particles are quite free and disengaged from each other, and

and are not diſturbed or conſtrained in their natural ſtate by mutual action, they will be at liberty, at leaſt for ſome time, to preſerve their motion in the direction in which it is impreſſed. Conſequently, bodies having the ſame tranſverſe ſection, but different ſurfaces, expoſed to the particles of the fluid, will be differently retarded, ſince the forces, in a direction perpendicular to that ſection, which meaſure the reſpective degrees of retardation which they meet with, will be different, as they correſpond to theſe ſeveral ſurfaces. Accordingly, Sir Iſaac Newton has demonſtrated, in the third propoſition, that, in ſuch a fluid, a cylinder would be twice as much reſiſted, as a globe of the ſame diameter. And, in the fourth, he ſhows, that, if the particles of the medium fly or rebound from the body with perfect elaſticity, a globe will ſuffer a reſiſtance, which bears to the force which would generate or deſtroy its whole motion, in the time it would move through a ſpace equal to two thirds of its diameter, the ratio of the denſity of the medium to the denſity of the globe; that, if the

particles are not at all reflected, the resistance to the globe will be twice as small as in the former case; and that, if they be reflected with any degree of elasticity, between that which is perfect and nothing, the resistance to the globe will be in the same intermediate ratio between the resistance in the first case, and that in the second. He likewise infers, by way of corollary, that, in a fluid so constituted, the resistance to a globe is in a ratio, compounded of the duplicate ratio of the velocity, the duplicate ratio of the diameter, and the ratio of the density of the medium. Whence, since the resistance of the medium must then be, *caeteris paribus*, in the duplicate ratio of the velocity, he shows a method of expressing the motion and resistance of the globe, by the space between the asymptotes and curve of an hyperbola.

Having thus considered the resistance arising from a discontinued fluid, or such as has not its particles contiguous, but freely arranged at equal distances from each other, and

and ſhown, that the reſiſtance to a cylinder, in a fluid of that nature, is double that to a globe of the ſame diameter, he proceeds to examine continued or compreſſed mediums, of a perfectly fluid nature, and, of courſe, void of tenacity, and likewiſe friction, when perfectly poliſhed bodies move in them.

Now, if a body placed in a medium, compreſſed to any certain degree, be forced to leave its place, the particles round the hinder part of it, as ſoon as it begins to move, will ruſh in to ſupply its place, and to prevent any vacuity or void, with that degree of velocity, which the ſaid compreſſion would communicate to them; and the particles ſurrounding the foremoſt part of the body, and receiving its preſſure, will have a tendency to move towards thoſe parts, the others left, in order to reſtore the aequilibrium. And, as the particles which receive the preſſure of the moving body, immediately preſs upon thoſe contiguous to them, and thoſe on others, and ſo on, the progreſſive motion of the fluid

fluid will, in this caſe, be leſs, and conſequently the reſiſtance depending thereon, than in a diſcontinued fluid, where the particles are quite diſengaged and independent on each other. But that motion which the preſſure ariſing from the compreſſion of the fluid occaſions, will depend upon the quickneſs with which the parts of the fluid ruſh in towards the place continually, which the body attempts to leave, and the celerity and readineſs with which this preſſure is propagated; and it will always be the leſs, the quicker this propagation is. Now, if the compreſſion be ſuppoſed indefinitely great, and the fluid devoid of elaſticity, this preſſure will be propagated inſtantaneouſly, and therefore can produce no motion, or change of motion, in the parts of the fluid; and, of conſequence, can neither increaſe nor diminiſh the reſiſtance which ariſes from the former motion. For the inſtantaneous propagation of this preſſure renders the action of the fluid depending on its compreſſion, the

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ſame on the parts of the body, before and behind.

Sir Iſaac has conſidered very fully the caſe of a body moving in ſuch an infinitely compreſſed non-elaſtic fluid, and has accordingly ſhown, in the ſixth propoſition of this ſection, that the reſiſtance to a cylinder in a fluid thus conſtituted, is four times leſs than the reſiſtance it would meet with in a diſcontinued non-elaſtic fluid of the ſame denſity. Conſequently it is only one eighth part of the reſiſtance of a diſcontinued fluid of the ſame denſity, provided it be perfectly elaſtic.

But he has not only demonſtrated, that theſe two ſorts of fluids, namely the diſcontinued and continued non-elaſtic fluids, give different degrees of reſiſtance to bodies moving in them, but that they likewiſe reſiſt very differently bodies which have the ſame tranſverſe ſection, but ſtrike the particles with different ſurfaces. For he has ſhown, that all bodies, whatever ſurfaces they preſs

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ſuch a continued medium with in paſſing through it, if they have the ſame or equal tranſverſe ſections, will be equally reſiſted. Whence, in his ſeventh propoſition, he infers, that the reſiſtance to a globe, moving uniformly in ſuch a medium, bears to the force which would generate or deſtroy the whole of its motion in the time it would move through a ſpace equal to eight thirds of its diameter, the ratio of the denſity of the fluid to the denſity of the globe.

In the firſt corollary he draws from this propoſition, he obſerves, that the reſiſtance to a globe in an infinitely compreſſed medium of this ſort, is in the ratio compounded of the duplicate ratio of the velocity, the duplicate ratio of the diameter, and the ratio of the denſity of the medium; that is, *caeteris paribus*, in the duplicate ratio of the velocity. In the ſecond, he informs us, that the greateſt velocity with which a globe can deſcend in ſuch a reſiſting medium, by the force of its own comparative weight, is that which it would acquire by falling *in vacuo* with the ſame weight, through a ſpace,

ſpace, which is to four thirds of its diameter, as the denſity of the globe to that of the fluid. The third corollary he derives from it is this, that, having the denſity and velocity of the globe at the beginning of its motion, with the denſity of the compreſſed fluid, the velocity and reſiſtance of the globe, and the ſpace deſcribed by it, correſponding to any time are given. And the laſt inference he draws from it, is this, that a globe, by moving in a compreſſed fluid at reſt, of the ſame denſity with itſelf, will looſe one half of its whole motion before it has paſſed through the length of two of its own diameters.

In the laſt propoſition of this ſection, he compares the theory for the reſiſtance to a globe, deſcending in ſuch an infinitely compreſſed and perfectly fluid medium, as I have been now ſpeaking of, with experiments made in water and air. Since ſuch a fluid reſiſts in the duplicate ratio of the velocity, and he had conſidered, as I have already obſerved, the aſcent and deſcent of a body in

a ſimilar or uniform medium, reſiſting, according to this law, in a direction perpendicular to the plane of the horizon, and urged by an uniform gravity, in the fifth propoſition of the ſecond ſection; he forms a calculus thereupon in this propoſition, with regard to falling bodies, and compares the concluſion with experiments.

And now that I am mentioning this propoſition, I cannot help thinking myſelf, in ſome meaſure, called upon to do a piece of juſtice to the memory of Mr Benjamin Robins, who has deſerved a great deal of true and genuine ſcience, and to clear him of a miſtake, with which he is very unjuſtly and erroneouſly charged by Mr Muller, in the 39th page of the introduction to his treatiſe on artillery, ſecond edition, printed at London in the year 1768. After obſerving, at the bottom of the immediately preceeding page, that the greateſt velocity which a leaden bullet of three quarters of an inch diameter can poſſibly have, is that which uniformly continued would carry it through

395 feet per ſecond, he expreſſes his ſentiments in relation to Mr Robins's method of determining the velocities of ſhot, in the following words. 'Mr Robins thinks to 'prove, in his ſeventh problem, that the velocity of the foregoing leaden bullet is '1668 feet in a ſecond, which is more than 'four times greater than that above; and, 'what is more extraordinary, he pretends to have found the ſame velocity by 'experiments. As he ſeems to build his 'theory upon Sir Iſaac Newton's principles, 'had he read the 40th propoſition, book 'II. of his Principia, he muſt have been 'convinced of his miſtake.' This is a very heavy charge, and, if it were well grounded, it would in a great meaſure invalidate all that Mr Robins has advanced, with regard to the determination of the firſt velocities of ſhot. For this is the only example that Mr Robins calculates in his ſeventh propoſition, from his theory of the elaſtic force of the fluid, generated by the firing of gun-powder, which he compares in his ninth propoſition, with the velocity aſcertain-

ed by means of his Balliſtic pendulum, and finds to be almoſt the ſame. This is the place that Mr Muller refers to, when he ſpeaks of Mr Robins, as pretending to have found the ſame velocity by experiments. Beſides, it muſt be obſerved, that Mr Robins does not ſpeak of this velocity as the greateſt that can poſſibly be communicated to ſuch a ball; for, undoubtedly, a much greater velocity may. Neither can he be ſaid to have built this part of his theory, at leaſt, in the ſmalleſt degree upon Sir Iſaac Newton's principles. For his method of determining the firſt velocities of bullets is entirely his own. But to come directly to the point. It appears abundantly plain to me, from the accuſation itſelf, that Mr Muller had not read, at leaſt with proper attention, the ſaid 40th propoſition of the ſecond book of the Principia. It relates intirely to the deſcent of bodies in an infinitely compreſſed and perfectly fluid medium; whereas, Mr Robins's calculation, juſt now mentioned, refers expreſsly to projectile motion. And that projectile velocity may be much greater than the greateſt velocity

velocity with which a body can defcend by its comparative weight, in fuch a fluid, is what no perfon who has confidered the fubject, with the leaft attention, will offer to deny. Even Galileo himfelf, who was the firft that ftudied with any care the motions of projectiles, and the nature of refifting mediums, feems to have been fufficiently fenfible of this truth. For, in his fourth dialogue, after obferving that the heavieft bodies would in time, by defcending in the air, acquire a degree of velocity, which afterwards could not be increafed, in the fame manner, as we fee light bodies foon arrive at their greateft velocity poffible, he fays, 'This 'determinate and ultimate velocity, may be 'called the greateft, which fuch a heavy body can naturally obtain in the air. But 'this velocity, I imagine, to be much lefs 'than that which is given to the fame ball 'flung by fired powder.' And he propofes to prove this, by firft firing a mufket loaded with a leaden ball, and a proper charge of powder, from an hundred yards high, or more, againft a ftone pavement, and then

then firing the ſame with a like charge againſt a ſtone of the ſame ſort, at a few yards diſtance, and examining which of the balls was moſt flatted. Every body now knows, that this imagination of his was perfectly juſt.

Sir Iſaac Newton begins the propoſition already quoted, with a rule for determining the greateſt velocity, with which a globe can deſcend, in an infinitely compreſſed and completely fluid medium; which take in his own words. 'Sit A pondus globi, B pon-'dus ejus in vacuo, D diameter globi, F 'ſpatium, quod ſit ad $\frac{4}{3}$ D, ut denſitas globi 'ad denſitatem medii, id eſt, ut A ad A—B; 'G tempus quo globus pondere B abſ-'que reſiſtentia cadendo deſcribit ſpatium 'F, et H velocitas quam globus hocce caſu 'ſuo acquirit. Et erit H velocitas maxima, 'quacum globus, pondere ſuo B, in medio 'reſiſtente poteſt deſcendere.' This, however, is the very rule which Mr Muller has made uſe of to determine the greateſt poſſible projectile velocity of a bullet of three quarters of an inch diameter, as I find by calculation,

calculation. If he has not taken the rule from this propoſition itſelf, but found it in ſome other place, as in the ſecond corollary of the immediately preceding propoſition except one, or picked it up at ſecond hand, one might eaſily account for his miſtake, by his barely looking at the enunciation itſelf, in which the word *progredientis* is uſed inſtead of *deſcendentis*; a circumſtance that would naturally lead one, who went no farther, to conclude, that it related more to projectile progreſſive motion, than to that of deſcent.

Theſe obſervations, I thought it incumbent on me to make, not only in vindication of Mr Robins, but in order alſo to prevent thoſe, who ſhall happen to peruſe the above-cited part of Mr Muller's performance, from ſwallowing ſuch a groſs and dangerous miſtake as he has fallen into, in conſtructing the table which he there gives, with regard to the greateſt poſſible velocities of ſhot.

Sir

Sir Iſaac Newton concluded, from ſome experiments which he made with globes falling from the higheſt part of the cupola of St. Paul's church in London, that the reſiſtance of the air to bodies moving in it, is nearly in the duplicate ratio of the velocity. And it is no wonder, that to ſuch ſmall velocities as theſe bodies muſt have deſcended with, it ſhould nearly be ſo, ſince the compreſſion of that fluid, which ariſes from the incumbent weight of the higher parts of the atmoſphere, is ſufficient to communicate to the particles of it a much greater degree of velocity. But, as the velocity of the moveable approaches nearer and nearer to an equality with that, which the parts of the fluid have, in conſequence of its compreſſion, the reſiſtance becomes greater than in this ratio. And when the former of theſe velocities is equal to, or greater than the latter, the reſiſtance becomes conſiderably greater than in the duplicate ratio of the velocity; the bullet then leaving a ſort of vacuum behind it, and ſuſtaining on its forepart the preſſure of a column of the atmoſphere,

phere, besides the resistance arising from the motion which it communicates to the parts of the fluid, that formerly, by rushing into the deserted place, balanced in some measure its resistance, and the more so, as the velocity of the particles of the fluid, arising from its compression, exceeded that of the moveable. Sir Isaac Newton, as I formerly observed, did not consider the case of a body moving in a fluid, which resists in the duplicate ratio of the velocity, when projected in any angle with the horizon. Mr John Bernoulli, however, has shown how this motion may be assigned by means of certain curves. But these curves do not fall under the known expressions for quadratures. His solution may be seen in the continued commentary which the two Jesuits, Le Seur and Le Jacquier, have written on the Principia. After examining it fully, in their commentary on the second section of the second book of that performance, they make the following remark. 'Ex quibus manifes-'tum sit, verae trajectoriæ descriptionem 'adeo perplexam esse, ut ex illa vix quid-

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'quam ad uſus philoſophicos aut mechanicos 'accommodatum poſſit deduci.' The ſame obſervation may be extended to a ſolution of the ſame problem, by Mr Euler, which I have ſeen in the Memoirs of the royal academy of ſciences at Berlin, vol. 9. page 321. He ſeparates his concluſion into an indefinite number of different claſſes of curves, accommodated to the different firſt velocities of bullets. But, from his ſolution, ſcarce any advantage can accrue to the practice of artillery. For, were it even reducible to common purpoſes, which it is not, it could only be applied to caſes of ſmall velocity. Sir Iſaac Newton appears to have been ſufficiently appriſed of the augmentation of reſiſtance above that of the duplicate ratio, which would take place, when the velocities of projectiles become very great, as we learn from the ſame 40th propoſition, cited already oftner than once, where, after mentioning the diminution of preſſure on the parts of the bullet behind, in conſequence of the increaſe of its velocity, and obſerving, that the compreſſion of the aerial fluid is not increa-

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ſed in the duplicate ratio of the velocity, which would be neceſſary to render the reſiſtance accurately in that ratio, he ſays, 'Globi velociores paulo minus premuntur a 'tergo, et defectu preſſionis, reſiſtentia eorum 'ſit paulo major quam in duplicata ratione 'velocitatis.' But it does not ſeem at all probable, that he had the leaſt ſuſpicion, that the reſiſtance would increaſe ſo prodigiouſly in the air, beyond this law of the duplicate ratio, to very ſwift velocities, as Mr Robins afterwards found it actually to do in experiment.

Even granting, however, that this law of reſiſtance ſufficiently comprehended that of the air in its full extent, and that it received no increaſe from an increaſe of velocity, I cannot underſtand the reaſon, why both mathematical and military people, for ſome time after Sir Iſaac publiſhed his Principia, imagined the parabolic theory, originally given by Galileo, ſufficient for the purpoſes of artillery, unleſs it be, that men at firſt paid more attention to the other parts of that

wonderful performance, which are much more easily comprehended, and relate immediately to much more popular and sublime subjects than the resistance of mediums, such as the theory of the tides, the theory of comets, the precession of the equinoxes, the theory of the moon, the figures, motions, and other circumstances relating to the bodies in the planetary system.

That this author's theory of resistance cost him a great deal of thought and attention, is evident from this, that he had been making experiments to ascertain the resistance of fluids, before he had been able to form any theory which he might examine by them. For, in the same proposition, he says. 'Experimenta hactenus descripta coepi, ut investigarem resistentias fluidorum, antequam theoria, in propositionibus proxime precedentibus, exposita, mihi innotesceret.' And, towards the end of it, he illustrates the seventh corollary to proposition 35th, by showing, how the diminution of progressive motion, which a globe projected in any medium

medium would ſuffer in a given time, on the ſuppoſition that the medium reſiſts nearly in the duplicate ratio of the velocity, may be computed. This progreſſive motion he might perhaps have had in view, when he put the word *progredientis*, in the enunciation of the propoſition, which probably led Mr Muller into his miſtake.

The ingenious Mr Benjamin Robins, who died at Fort St. David in the year 1751, in the character of engineer general to the Eaſt India company, applied himſelf very early to the making of experiments in gunnery, which he conducted with a great deal of judgement and ſucceſs. In 1742, he publiſhed a ſmall treatiſe in two chapters, to which he gave the title of New principles of gunnery. His principal intention in this performance is, to aſcertain the velocities of bullets, particularly their firſt velocities, and to demonſtrate that the reſiſtance of the air to ſuch ſwift motions is much greater than any perſon before his time had imagined.

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In the firſt chapter, he propoſes two methods, which are intirely his own, of determining the firſt velocities of military projectiles. One of theſe is by meaſuring the force of a permanent elaſtic fluid, generated by the firing of gun-powder, made according to the government-ſtandard, in which it was lodged before the exploſion, in a fixed ſtate. This fluid, he ſays, agrees intirely with the air, in the articles of condenſation, expanſion, and elaſticity. The other is by firing againſt a pendulum, which ſwings freely, and has a narrow ribbon faſtened to its bottom, which, by paſſing between two ſteel edges, always meaſures the chord of the arc deſcribed by the motion of the pendulum. From theſe two methods he brings out the velocities of balls of the ſame diameter, when fired with equal quantities of powder of the ſame quality, in a great variety of inſtances, almoſt in a perfect ſtate of agreement or equality, which is a circumſtance that makes much for the truth of both.

In the ſecond chapter, he ſhows by experiment,

periment, that the reſiſtance of the air, even to a velocity of 400 feet per ſecond, is ſomewhat greater than in the duplicate ratio of the velocity; and that, to the ſwifter motions of muſket or cannon-ſhot, the reſiſtance exceeds this ratio, nearly in the ratio of 3 to 1. He likewiſe was the firſt who diſcovered, that a bullet fired from an ordinary muſket or cannon, beſides being affected by the reſiſtance of the air, and the action of gravity, receives a whirling motion, or rotation round an axis, the poſition of which is not at all conſtant, but uncertain and variable; and that this rotatory motion is the real cauſe, why the track of the ball is doubly incurvated, and carries it to a conſiderable diſtance, from the plane paſſing through the axis of the piece perpendicularly to the horizon. This lateral deflection or deviation from the incurvated line, which the bullet deſcribes in paſſing through the medium by the action of gravity alone, very much diſtreſſed Mr Robins in making his experiments. The exiſtence of this motion he clearly demonſtrates; the difficul-

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ty he acknowledges, and ſeems abundantly ſenſible, that it muſt introduce a degree of uncertainty into all concluſions drawn from experiments made with pieces generally in uſe.

This judicious experimenter does not pretend to aſcertain accurately the law of reſiſtance in the air, or to determine preciſely, how the augmentations to that, which is in the duplicate ratio of the velocity, take place. His whole treatiſe, however, is written with a great deal of perſpicuity and elegance, and well deſerves to be peruſed and examined attentively by every engineer and artilleriſt.

In a diſcourſe which was afterwards read before the royal ſociety, and is inſerted at page 179. vol. I. of his mathematical tracts, publiſhed at London by Mr Wilſon, to whom, with Martin Folkes, Eſq; then preſident of the royal ſociety, he left by his laſt will the care of his papers, he mentions a very curious circumſtance, with regard

gard to the reſiſtance of the air. Having obſerved, in the courſe of his experiments, that there is a certain velocity, at which the augmentation of reſiſtance is prodigiouſly increaſed, and finding this velocity to coincide nearly with that, which ſound has in its propagation through the air, he offers his reaſons for believing, that, in this caſe, the air does not make its vibrations quick enough to fill up inſtantaneouſly the place deſerted by the bullet, and therefore leaves a ſort of vacuum behind it. He ſays, that the reſiſtence which the body then meets with, is thrice as great as that, which is computed according to the duplicate ratio of the velocity. This, however, he propoſes, to be tried more accurately by experiment; and does not pretend to ſay, that this treble reſiſtance abſolutely takes place inſtantaneouſly, or all at once. The very experiments made by himſelf are ſufficient to overturn this ſuppoſition. And I have great reaſon to believe, as I ſhall, on ſome other occaſion, take an opportunity of ſhewing, that ſuch is not in reality the caſe, but that there is a certain

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gradation in this increaſe of reſiſtance, which may be determined by calculation, and confirmed by experiment. In this diſcourſe, Mr Robins lays down two propoſitions, upon which he proceeds in approximating the actual ranges of pieces with ſmall angles of elevation, ſuch as do not exceed 8° or 10°, which he ſets down in a table compared with their correſponding potential ranges. The firſt of theſe is, 'That, till the velocity 'of the projectile ſurpaſſes that of 1100 feet 'in a ſecond, the reſiſtance may be eſteemed 'to be in the duplicate proportion of the 'velocity.' And the ſecond is, 'That, if 'the velocity be greater than that of 11 or '1200 feet in a ſecond, the abſolute quan'tity of the reſiſtance will be near three 'times as great, as it ſhould be by a com'pariſon with the ſmaller velocities.' Neither of theſe rules, Mr Robins ſeems to be ſatisfied, is accurately or ſtrictly true. He only lays down theſe poſitions as calculated to be of conſiderable ſervice in the practice of artillery, till a more complete and accu-

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rate theory of resistance, and the changes of its augmentation, be obtained.

I am certain, at least from some experiments which I saw tried in company with two gentlemen of my acquaintance, with a rifled field-piece, cast at Carron, for some experimental purposes *, which had a quadrant adapted to it, with a telescope, having the plane passing through its axis, perpen-

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* This invention was the united contrivance of Dr Lind and Captain Blair of the 69th regiment of foot, two gentlemen well known to their acquaintance for their ingenuity. To such a rifled piece carrying a leaden ball, with protuberances or knobs cast on it to fit exactly the rifles, a quadrant was adapted with a telescope, cross wires, &c. This quadrant is equally applicable to any gun of a similar construction, of whatever size or caliber. The line of collimation of the telescope being once adjusted, so as to be truly parallel to the axis of that, or any other such gun, when the quadrant is set at o degrees; if the quadrant be afterwards set to any degree of elevation, and the gun be moved so as to bring the intersection of the cross-wires on the object to be fired at, the bore of the gun will then have the same elevation above it, and the plane passing through its axis will be parallel to that passing through the axis of the telescope, whatever position the carriage of the gun happens to stand in,

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dicularly to the horizon, parallel to that passing through the axis of the bore of the piece perpendicularly to the same: That the resistance of the air to a velocity somewhat less than that mentioned in the first of these propositions is considerably greater than in the duplicate ratio of the velocity; and that, to a celerity somewhat greater than that condescended on in the second, the resistance is a good deal less than that which is treble the resistance in the said ratio. Indeed,

This will serve to give a general idea of the piece made use of; but a particular description of all its parts I omit inserting here, as these gentlemen have not yet thought proper to communicate it to the public themselves.

It ought likewise to be observed, that all balls used for rifled pieces should be made of lead; for balls made of cast iron would soon destroy the rifles. Besides, as lead is specifically heavier than cast metal, it will meet with a smaller degree of retardation from the resistance of the air, and will consequently fly to a greater distance. There is also another advantage arising from the use of leaden balls with small field-pieces, which is this; that it frees an army from the necessity of carrying large quantities of iron-shot from place to place, since lead is commonly to be got in most countries from the roofs of houses and churches, and may be cast into balls as occasion requires.

deed, ſome of Mr Robins's own experiments ſeem neceſſarily to make it ſo; ſince, to a velocity no quicker than 400 feet in a ſecond, he found the reſiſtance to be ſomething greater than in that ratio. After aſcertaining the firſt velocities of the bullets with as much accuracy as poſſible, I inſtituted a calculus, from principles, which had been lying by me for ſome time before, and found the reſiſtance to approach nearer to that which exceeds the reſiſtance in the duplicate ratio of the velocity by that, which is in the ratio of the velocity, than to that, which is only in the duplicate ratio. However, I have not had an opportunity of ſeeing ſo many experiments of this ſort tried, as I would judge ſufficient to enable me to pronounce poſitively with regard to the accurate law of the air's reſiſtance.

It is well known, that Sir Iſaac Newton has not conſidered the motion of a body projected in any angle with the horizon in a fluid, which reſiſts more than in the duplicate ratio of the velocity, by a reſiſtance

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in the ratio of the velocity, and acted upon during its flight by gravity. And I do not know that any author has confidered it in fuch a manner, as to affift any one in forming a calculus, or computation, with refpect to it. After I fhall have got, however, a fufficient number of experiments to my fatisfaction, and fuch as I can depend upon as being perfectly accurate, I fhall probably lay down the mathematical calculations both for this law of refiftance, and for feveral others as yet not examined by any perfon, fome of which I ftrongly fufpect, from different appearances, and, I think, with good reafon, approach exceedingly near, if not entirely, to the real refiftance of the air under different circumftances; and fhall likewife illuftrate the principal propofitions in the fecond book of the Principia, independent, in a great meafure, of the hyperbola, being already in poffeffion of the chief materials requifite for that undertaking. And I believe, that the frequent ufe made of this figure by Sir Ifaac Newton in that book, is

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one of the principal things that freighten moſt people from reading it.

I muſt, however, obſerve, that, in the experiments mentioned above, the lateral deflection or deviation, ſo much complained of by Mr Robins in different parts of his tracts, was entirely prevented. For this departure from the plane paſſing through the centre of the teleſcope, and that of the mark on the canvas ſeen through it, at the diſtance of 1500 feet, did not exceed, when greateſt, one foot and an half, or two feet, which might eaſily be occaſioned by the minuteſt change in the poſition of the quadrant, except one morning when the wind was unfavourable, and blew pretty hard, and thereby ſent the bullet to the diſtance of about a yard from that plane. The balls were all numbered on the faces which were ſent home to the breach of the gun, had knobs or protuberances upon them to anſwer the grooves or rifles ; and, of upwards of forty of them fired to the diſtance of 1500 feet, not one had ſhifted its face, the numbered ſides

ſides being all entire, and the oppoſite ones conſiderably bruiſed and deformed. Had Mr Robins tried that great multiplicity of experiments which he made, with ſuch a piece as this, he would probably have been even more ſucceſsful than he was in diſcovering the nature of the air's reſiſtance.

In the ſame diſcourſe already mentioned, he gives directions for approximating the actual range at any angle not exceeding 45°, the potential range being known, and forms a table for comparing the potential randoms with the actual ranges at an angle of 45°, on the ſuppoſition that the propoſitions I ſpoke of as laid down by him are true. The reſt of his tracts on gunnery, which relate to ſeveral different particulars, deſerve to be read with great attention. His practical maxims ought to be remembered, and ſtudied carefully by every artilleriſt. At the end of them, there is a diſſertation of his, on the Nature and Advantage of Rifled Barrel Pieces, wherein he mentions ſome experiments made by him with pieces of this ſort,

in which the deflection of the bullets was prevented. This, he says, was owing to the whirling motion, which they received from the rifles, round an axis coinciding with the lines of their flight. He observes, that this prevention will always take place, when the angle of elevation is not great, and the path of the bullet not much incurvated. However, if either the angle of elevation, or the curvature of the bullet's tract through the air be great, the inclination of the axis round which it whirls will cause irregularities, which will often produce considerable deflections.

Accordingly, he proposes to make use of bullets of an egg-like form, instead of spherical ones, and to fire them with their broad ends foremost, that their axes may be always carried by their centres of gravity into the lines of their flight. And no doubt such a proposal might be very successfully executed. In making these experiments, however, Mr Robins did not use the contrivance of knobs or protuberances, mentioned above,

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on his bullets, which might have exactly filled the rifles, without preventing the bullets themselves from being easily sent home in the piece *. All experiments in gunnery for ascertaining the resistance of the air, or the force of gun-powder, should unquestionably be made with rifled pieces, properly adjusted, and furnished with some such contrivances as those described above. For those which are made, even with the best pieces now in common use, cannot be much depended on. And it cannot be doubted, that, were the resistance of the air accurately ascertained, and easy rules delivered for computing it, the introduction of rifled pieces, with such contrivances, in war, would turn the advantage greatly on the side of those who first should use them. Besides, such pieces

* There is one circumstance which ought likewise to be particularly attended to, which is this; that each bullet retained the whirling motion which it received from the rifles, during the whole course of its flight. For the bruised face of each bullet was twisted in such a way, in entering the ground, as put this fact beyond all doubt.

pieces well adjusted, would, I think, greatly facilitate the determination of the first velocities of projectiles, which is the first desideratum in gunnery. Both Mr Robins's methods are certainly very ingenious, and discover the author of them to have been a very able and sagacious man. But the first of them is necessarily liable to great uncertainty, from the changes produced in powder by heat, moisture, and other circumstances. And the second is scarce applicable, at all, to large or heavy shot.

However, having rifled pieces properly adjusted, I could undertake to calculate such tables on the supposition of any law of resistance whatever, that, from them, the first velocities of all bullets might be taken, with almost as much ease, as we usually take out logarithms; and that too, on any occasion, and in any place, where such pieces can be fired. And it ought not to be omitted, that there is one very necessary circumstance, which should be particularly attended to in performing experiments in gunnery, which

is this, that no perſon ſhould ſet about making ſuch experiments, who is not poſſeſſed of a ſufficient ſhare of mathematical knowledge, to conduct him in his proceedings, and to enable him to apply accurate meaſurement and calculation to thoſe ſuppoſitions, which his own ſagacity, whetted by ſuch knowledge, may ſuggeſt to him in the courſe of trial and obſervation. A man, who is not thus qualified, may perform millions of experiments, without deriving from them much information to himſelf, or aſſiſtance to others.

As I do not know of any author ſince Mr Robins's time, who has made any diſcovery worthy of notice, relating to the reſiſtance of the air, I ſhall ſet down ſome problems, or queſtions, in reference to the reſiſtance of mediums, to exerciſe in the mean time the ingenuity of mathematicians and ſcientific engineers, who are above tying themſelves down to the erroneous practices of their predeceſſors and companions. Moſt of them I have

have already confidered, and the reft I fhall probably make the fubject of my future examination.

PROBLEM I.

To determine the firft velocity of any bullet.

PROB. II.

To determine the afcent and defcent of a body in a medium which refifts in the ratio of the velocity, when it is acted upon by a gravity, which increafes as the fquare of the diftance from the centre of gravitation decreafes.

PROB. III.

To determine the motion of a body projected in a fuch medium, and acted upon by the fame fort of gravity.

PROB.

PROB. IV.

To determine the velocity, which a body muſt acquire, by falling from reſt in ſuch a medium, in order to ſuffer a degree of reſiſtance, equal to the force which it receives from the ſame ſort of gravity; and when,

PROB. V.

To determine the ſame things in ſuch a medium, when the body is acted upon by a gravity, which is in any ratio, either inverſely or directly, of the diſtance from the centre of gravitation.

PROB. VI.

To determine the motion of a body projected in ſuch a medium, and acted upon by a gravity in any ratio, either inverſely or directly, of the diſtance from the centre of gravitation.

PROB.

PROB. VII.

To ſolve the five immediately preceeding problems, when they relate to a medium, which reſiſts partly in the ratio of the velocity, and partly in a conſtant ratio.

PROB. VIII.

To ſolve each of the ſame problems, when they relate to a medium, which reſiſts in the ratio of any multiple of the velocity.

PROB. IX.

To ſolve each of the ſame, when they relate to a medium, which reſiſts partly in the ratio of any multiple of the velocity, and partly in a conſtant ratio.

PROB. X.

To ſolve each of the ſame problems, when they relate to a medium, which reſiſts in ſuch

ſuch a multiple of the velocity, as increaſes arithmetically to a certain limit, and then becomes conſtant or invariable.

PROB. XI.

To ſolve each of the ſame, when they relate to a medium, which reſiſts in ſuch a multiple of the velocity, as increaſes geometrically to a certain limit, and then becomes invariable.

PROB. XII.

To ſolve each of the ſame, when they relate to a medium, which reſiſts in ſuch a multiple of the velocity, as decreaſes, either arithmetically or geometrically, to a certain limit, and then becomes invariable.

PROB. XIII.

To ſolve each of the ſame, when they relate to a medium, which reſiſts partly in ſuch a multiple of the velocity as increaſes,

either

either arithmetically or geometrically to a certain limit, and then becomes invariable, and partly in a conſtant ratio.

PROB. XIV.

To ſolve each of the ſame, when they relate to a medium, which reſiſts in ſuch a multiple of any multiple of the velocity, as increaſes, either arithmetically or geometrically, to a certain limit, and then becomes invariable.

PROB. XV.

To ſolve each of the ſame, when they relate to a medium, which reſiſts partly in ſuch a multiple of any multiple of the velocity, as increaſes, either arithmetically or geometrically, to a certain limit, and then becomes invariable, and partly in a conſtant ratio.

PROB. XVI.

To ſolve each of the ſame, when they relate

late to a medium, which resists partly in such a multiple of the velocity, as decreases, either arithmetically or geometrically, to a certain limit, and then becomes invariable, and partly in a constant ratio.

PROB. XVII.

To solve each of the same, when they relate to a medium, which resists in such a multiple of any multiple of the velocity, as decreases, either arithmetically or geometrically, to a certain limit, and then becomes invariable.

PROB. XVIII.

To solve each of the same, when they relate to a medium, which resists partly in such a multiple of any multiple of the velocity, as decreases, either arithmetically or geometrically, to a certain limit, and then becomes invariable, and partly in a constant ratio.

PROB. XIX.

To ſolve each of the ſame, when they relate to a medium, which reſiſts in any multiplicate ratio whatever of the velocity.

PROB. XX.

To ſolve each of the ſame, when they relate to a medium, which reſiſts in ſuch a multiple of any multiplicate ratio of the velocity, as increaſes or decreaſes arithmetically or geometrically, to a certain limit, and then becomes invariable.

If the multiple of the ratio of the velocity be (2), and the multiple increaſing to a certain limit be an arithmetical progreſſion, the law of reſiſtance will, I apprehend, be found to differ not very widely from that which really takes place in the air.

PROB. XXI.

To ſolve each of the ſame, when they re-

late to a medium, which reſiſts more than in a multiplicate ratio of the velocity, by a reſiſtance, which is in the ratio of the velocity.

PROB. XXII.

To ſolve each of the ſame, when they relate to a medium, which reſiſts more than in a multiplicate ratio of the velocity, by a reſiſtance which is in the ratio of the velocity, and likewiſe by a reſiſtance which is in a conſtant ratio.

PROB. XXIII.

To ſolve each of the ſame, when they relate to a medium, which reſiſts more than in ſuch a multiple of a multiplicate ratio of the velocity, as increaſes or decreaſes arithmetically or geometrically, to a certain limit, and then becomes invariable, by a reſiſtance, which is in the ratio of the velocity.

If the increaſing multiple be an arithmetical

tical ſeries; and the multiple of the ratio of the velocity (2), the law of reſiſtance will approach nearer to that of the air, than the law mentioned after problem 20.

PROB. XXIV.

To ſolve each of the ſame, when they relate to a medium, which reſiſts more than in ſuch a multiple of a multiplicate ratio of the velocity, as increaſes or decreaſes arithmetically or geometrically to a certain limit, and then becomes invariable, by a reſiſtance, which is in the ratio of the velocity, and a reſiſtence, which is in a conſtant ratio.

If the increaſing multiple be an arithmetical ſeries, and the multiple of the ratio of the velocity (2), the law of reſiſtance will approach exceedingly near to that which actually holds in the air.

Thus have I endeavoured to enumerate, with as much brevity as poſſible, the opinions of the moſt ſagacious and intelligent authors,

thors, with regard to the reſiſtance of mediums, the advances which they have made towards the aſcertaining of the air's reſiſtance to bodies of different forms, moving in it with different degrees of velocity, the artifices which they have employed, in order to derive ſome uſeful concluſions, from their notions of this reſiſtance, in the practice of gunnery, and to point out what and how much is ſtill wanting for the completing of this art, both in theory and practice.

But what I have already obſerved, with regard to the reſiſtance of mediums, relates only to that which is made to the ſuperficies of bodies moving through them. And this indeed is the only ſort of reſiſtance, which ought to be taken into account by thoſe who treat of this ſubject, or conſider the doctrine of projectiles. Sir Iſaac Newton, in conſequence of an opinion, which prevailed very generally among the philoſophers of his time, that there is a certain very ſubtile aetherial medium, which paſſes through the pores and interſtices of all bodies, with great

great eafe and readinefs, condefcended to make an experiment, which, however, he never repeated, probably not thinking it worthy of notice, to try, whether bodies fuffer any refiftance on their internal parts from fuch a medium. For he conceived, that, if there be fuch a fluid, it muft give refiftance to the internal parts of bodies. The experiment is related at full length by himfelf in the general fcholium to Prop. 31. Book. 2.

But, from the nature of the experiment itfelf, as well as from the circumftances attending it, we have reafon to affirm, when we examine his conclufion, that no inference can be drawn from it, favourable to the fuppofition of the exiftence of a fluid, which refifts internally the parts of bodies moving through it. For he was only able to conclude, all other caufes being excluded, which exclufion he does not however fay ought to be made, that the internal refiftance to the parts of pieces of different metals put into a wooden box, fo as to fill it, which was made to fwing, like a pendulum, from

a

a point, diſtant only about ſix feet from the perpendicular, amounted to no more than the $\frac{1}{5928}$ part of the external reſiſtance ariſing from the air. Now we know, that the reſiſtance of the air to bodies moving in it with ſuch ſmall velocities, is but inconſiderable, and that the $\frac{1}{5928}$ part of it was ſufficiently ſmall, to have eluded intirely any meaſurement, which could have been derived from his experiment, particularly when we conſider the circumſtances which attended it. For he weighed the fir-box, into which theſe pieces of different metals were put, part of the thread which ſuſpended it, the quantity of air which it contained, before any thing was put into it, and compared the ſum of theſe weights with that of the box when filled with the different metals. He likewiſe contracted the thread, in conſequence of the increaſe of tenſion ariſing from the additional weight of the metals put into the box.

Now, the leaſt miſtake in relation to any of theſe circumſtances, or with regard to the point

point from which he allowed the box to ſwing when empty, was ſufficient to have brought out his concluſion. He might probably have drawn a quite different one, had he tried the experiment a ſecond time, which he has informed us he did not, thinking, it is likely, the ſubject of it too ridiculous to merit much attention. And indeed it muſt be owned, that it was an experiment ſcarce worthy of the great Newton. His words are theſe: 'Hoc experimentum recitavi memoriter; nam charta, in qua illud aliquando deſcripſeram, intercidit. Unde fractas quaſdam numerorum partes, quae memoria exciderunt, omittere compulſus ſum. Nam omnia denuo tentare non vacat.'

This aether has been imagined to be the cauſe of gravitation, coheſion, magnetiſm, repulſion, ſenſation, and of almoſt all the phaenomena in nature. It has been conceived of as growing always denſer, as you recede from the bodies of the ſun and planets.

But,

But, if the motion of the earth towards the ſun be occaſioned by the impulſes of a medium growing always denſer as you recede from that luminary, and its elaſtic force increaſe with its denſity, like that of the air, ought not theſe impulſes to be always diminiſhing as you go nearer to the ſun? But, they muſt be always increaſing, to produce gravitation. Wherefore the hypotheſis is abſurd.

In like manner, if this aether is rarer in, and at the planets, than at ſome diſtance from them, ought not the acceleration of bodies towards the earth to be always diminiſhing, inſtead of increaſing? This, however, we know, is not the caſe.

It will move in two oppoſite directions, in impelling the earth and moon towards their common center; in two other oppoſite directions, in impelling the ſun and earth towards their common center; in two other oppoſite directions, in impelling the ſun and each of the other planets toward their common

mon centres, respectively. And, as the position of the earth and moon is infinitely varied in the course of one revolution round the sun, does not this fluid move in diametrically opposite directions across the direction of its motion towards that body, and in an infinite variety of ways and positions? Is not the same observation applicable to Jupiter and his satellites, to Saturn and his satellites, and to all the bodies in the solar system, since they mutually gravitate towards one another? What an infinite variety of opposite motions, then, must this aether dance with through the universe? That such an infinite diversity of opposite motions in the waves or pulses of this fluid should exist, is altogether impossible. Would they not soon, by encountering, destroy one another, and reduce the whole fluid to an aequilibrium or state of rest?

Besides, as soon as the common center of the earth and moon has changed its position in absolute space, and come to some other point, the pulses or waves of this fluid must

proceed in oppoſite directions towards that point, and impel theſe bodies towards the ſame. And, as theſe bodies are connected with the other bodies in the ſyſtem, in all poſitions they may happen to be in, we ſhall have aetherial whirlpools in almoſt every point of ſpace bounding the ſolar ſyſtem. Let any one then, who chuſes to hold ſuch an abſurd hypotheſis, ſhow, how it is poſſible for a fluid to move through itſelf, in an infinite variety of oppoſite directions, at one and the ſame time.

But theſe are not one half of the abſurdities, that this hypotheſis is clogged with. The force of gravity, with which any body tends towards another, depends not only on its diſtance from, but alſo on the quantity of matter in, that other body. How then, is the elaſtic force of this medium, in impelling bodies towards one another, varied with their quantities of matter? The poſſibility of this, it is incumbent on thoſe to demonſtrate, who hold the exiſtence of ſuch an aether.

Beſides,

Besides, if such a fluid be the cause of gravitation, how can we be certain of the truth of this proposition, that the specific gravities of bodies, or their weights under equal bulks, are proportional to their quantities of matter. For how can the solid particles of this fluid equally impel the solid internal particles of bodies respectively, in the direction of gravity? This cannot be supposed by the advocates for such an aether, without maintaining the penetration of impenetrable substances, or showing matter not to be impenetrable, which has never yet been done, and is altogether inconsistent with the Newtonian idea of it.

And, even on the supposition, that the densest body we know is far from being perfectly solid, but has many pores or interstices between its constituent parts; and that the solid particles of this fluid, by entering these pores, could impel, in some direction or other, every particle of the body; those which were impelled in a direction perpendicular to that of gravity, would have no gravitation at all, and

and thoſe which received the impulſes of theſe aetherial particles, in directions any way inclined to that of gravitation, would have their gravities diminiſhed in the ratios of radius to the coſines of theſe angles of inclination reſpectively. They muſt either draw this concluſion, or maintain, that ſolid particles may be penetrated by ſolid particles, which laſt is an impoſſibility, and, of courſe, abſolute nonſenſe.

Again, let us ſuppoſe, that there were two plates or laminae, one of gold, and the other of cork, ſo thin, that the ſolid particles of a fluid thus conſtituted might eaſily impel each of the particles compoſing them, in the direction of gravity, without the abſurd hypotheſis of the compenetration of matter. According to the univerſally received idea of gravitation, theſe ought to deſcend equally towards the earth, with gravities proportional to the ſolid particles which they reſpectively contain. This then being the caſe, the number of ſolid particles in the fluid on the ſame ſurface muſt at leaſt

equal,

equal, if not exceed, the number of ſolid conſtituent particles of the plate of gold. Nay, as many of them muſt have paſſed through the interſtices between the conſtituent ſolid particles of the plate of cork, have we not reaſon to conclude, that many of them likewiſe paſs through the pores, between the ſolid particles compoſing the plate of gold? Be this, however, as it will, if the particles of the fluid be equal among themſelves, one of the ſolid component particles of the cork muſt be equal, in reſpect of matter, to one of the component particles of the gold. And the like inference would hold, in relation to all bodies and fluids that we are acquainted with. Now, if the particles of the fluid be equal to thoſe of the gold, as we have reaſon to infer from this concluſion, drawn in conſequence of an aether, it muſt at leaſt equal gold in denſity. In this caſe, the heavenly bodies, on the ſuppoſition, that their mean denſities equalled that of gold, would have one half of their whole motions deſtroyed, before they moved through ſpaces equal reſpectively to twice their diameters;

Princi-

Principia, book 2. prop. 38. cor. 4. and ſcholium to lemma 7. And, ſurely, it lies on the aſſertors of the exiſtence of the ſaid aether, if they refuſe this, to demonſtrate, that the particles compoſing it are leſs than thoſe compoſing gold.

But how can a fluid ſo conſtituted be the cauſe both of attraction and repulſion, which we obſerve to take place between the ſame bodies at different diſtances? Why ſhould it, at the ſame inſtant of time, give riſe both to gravitation and coheſion, when they act in direct oppoſition to each other? How can that which is ſuppoſed to be the cauſe of gravitation, be likewiſe referred to, as the cauſe of magnetiſm, which, from experiments made by Sir Iſaac Newton and others, does not ſeem to be ſubject to the ſame law with it; nor even, in different magnets, to one and the ſame law? If the elaſticity of this fluid be the cauſe of all theſe different phaenomina, what modifications does it undergo, to produce theſe different effects? For, I confeſs, that, unleſs in reſpect of degree, I can

can diſcover no alterations it could poſſibly be ſubject to. And the degree of power in any cauſe does not certainly change the nature, either of the cauſe, or the effects which it produces, but only varies the quantity or meaſure of it.

Beſides, ſuppoſing we were even to allow, that the elaſticity of this æther is the cauſe of all theſe things, which we cannot poſſibly do, without allowing manifeſt abſurdities and impoſſibilities, we might aſk, what is the great advantage or inſtruction, ariſing from deducing gravity in this manner, and what is the cauſe of this elaſticity? Is it a more general property in bodies than gravity? Surely, we have no reaſon to ſay ſo. We know of many bodies devoid almoſt of elaſticity, but of none, which do not poſſeſs gravity in proportion to their quantities of matter reſpectively. Is it a proper method of philoſophiſing, to account for a thing perfectly general, as far as we have yet been able to diſcover, by that which is only limited and particular? How could the motions

of

of the heavenly bodies, or indeed any regularity of motion in the universe, subsist with the existence of such a fluid?

Some assertors of this doctrine have, it is true, startled at the regular and undiminished motion of the celestial bodies, through so long a series of ages, and have affirmed this aetherial fluid of theirs not to be inert but devoid of resistance; and even Mr Maclaurin has supposed the existence of equal solidities, with unequal degrees of resistance, or *vis inertiae*, not adverting to this, that the supposition, which he makes partially, may with equal justice be made universally; and that with the same reason that it is supposed, that matter may exist with different degrees of *vis inertiae*, it may be supposed to exist without *vis inertiae* altogether. What is this, however, but to affirm, that matter may both be, and not be, at the same time? For, remove the idea of the *vis inertiae* of matter, and you likewise remove that of its solidity; and what idea have we of matter without solidity? It is impossible to have

any

to have any at all. Theſe perſons then, who make not this fluid inert, make it no longer matter, but in every reſpect a vacuum or empty ſpace.

After Sir Iſaac Newton, in conſidering the phyſical motions of the heavenly bodies, had ſo ſucceſsfully applied to them the doctrine of gravitation, joined with mathematical demonſtration, his principle of univerſal attraction was ſeverely attacked, particularly by the Carteſians, as an occult quality. The leap from a perfect plenum, to an abſolute vacuum, was great, and what he never could have brought a Carteſian philoſopher to have taken in company with him. He ſhowed the inſufficiency of their vortices to account for the celeſtial motions. But, as he was a man very much averſe from wrangling, he perhaps did not chuſe to deny them, intirely, the ſuppoſition of the poſſibility of ſome fine fluid or aether exiſting in the univerſe. However, ſo far was he from ſuppoſing it not to be inert, that he even condeſcended, as I have obſerved above,

to make an experiment, to aſcertain the internal reſiſtance which this fluid would neceſſarily give to bodies in paſſing through them. He found no reaſon, however, to conclude with certainty, that there is any ſuch reſiſtance to bodies; and accordingly he no where, in his admirable theory of reſiſtance, dwells upon it, or even takes it into conſideration.

Whether Sir Iſaac Newton, in the beginning of life, intirely diveſted himſelf of the abſurd notions and prejudices of his contemporary philoſophers, with regard to the exiſtence of ſome fluid matter or other, diffuſed and expanded through ſpace, I will not pretend to determine. But that he never was much delighted with the idea, is evident; that he laid no ſtreſs upon it, is equally evident; and that he at laſt intirely rejected it, as abſurd and ridiculous, and did not believe the exiſtence of ſuch a fluid, is almoſt demonſtrably certain from this circumſtance, that, to the edition of his Principia, publiſhed by Dr Pemberton, by his

own direction, and under his own eye, there is prefixed a preface, written by the very ingenious and accomplished Mr Coles, affirming gravity to be as much a primary quality of matter, as extension and impenetrability, and declaring, in express terms, the hypothesis of a celestial fluid to be quite absurd and untenable, and altogether unworthy of a philosopher. This excellent person, whose early death was an irreparable loss to the scientific world, after showing the existence of such a fluid to be inconsistent with the celestial motions, from the very theory of resistance, demonstrated by Sir Isaac Newton himself, in the second book of his Principia, concludes his observations in relation to it, with the following words. 'Itaque concludendum erit, fluidi 'coelestis nullam esse vim inertiae, cum 'nulla sit vis resistendi, nullam esse 'vim qua motus communicetur, cum 'nulla sit vis inertiae, nullam esse vim 'qua mutatio quaelibet vel corporibus singu-'lis vel pluribus inducatur, cum nulla sit vis 'qua motus communicetur; nullam esse om-

'nino

'nino efficaciam, cum nulla ſit facultas mu-'tationem quamlibet inducendi. Quidni 'ergo hanc hypotheſin, quae fundamento 'plane deſtituitur, quaequae naturae rerum 'explicandae ne minimum quidem inſervit, 'ineptiſſimam vocare liceat, et philoſopho prorſus indignam. Qui coelos materia fluida 'repletos eſſe volunt, hanc vero non inertem 'eſſe ſtatuunt, hi verbis tollunt vacuum, re-'ponunt.' Can there be any declaration more expreſs than this? And, had Sir Iſaac really believed the exiſtence of ſuch a fluid, would he ever have permitted Dr Pemberton to have prefixed to his principal work a preface, which expreſsly pronounces this his belief to be altogether deſtitute of foundation, and in every reſpect unworthy of a philoſopher: And, whether he believed it or not, as he has offered no arguments to prove it, but declares that he does not know what this aether is, the hypotheſis muſt be given up and renounced; ſince I have already ſhewn, that it is altogether inconſiſtent with phaenomena, and that innumerable arguments may be produced,

each

each of them almoſt ſufficient entirely to o-verturn it.

I ſhould willingly have ſaved myſelf the trouble of making theſe obſervations, with regard to a thing in itſelf ſo abſurd and ridiculous, had I not obſerved, that ſome perſons who never read the works of this great man, as it appears from their framing of ſuppoſitions, with regard to this aether, diametrically oppoſite to what he has advanced in the ſecond book of his Principia, where he treats of fluids in general, make uſe of his name and authority, to procure credit and reputation to ſome of the moſt wild and romantic theories in medicine, chemiſtry, and ſome branches of natural philoſophy, that ever were formed by a reaſonable mind, which they indeed derive from this imaginary ſource, that has no real exiſtence in nature. And, though I cannot pretend, in this place, to expoſe particularly ſuch abſurdities ; yet they ought to be held forth to the world in their true and genuine deformity, and to be ſeparated from the name and authority of a man who never gave them the ſmalleſt countenance.

[illegible] them almost sufficient entirely to [illegible]

I could willingly have saved myself the [illegible] of making these observations, with [illegible] to a thing in itself so absurd [illegible] institutions, had I not observed, that [illegible] persons who never read the works of [illegible] great man, as it appears from their ex[illegible] of suppositions, with regard to this [illegible] diametrically opposite to what he advanced in the second book of his [illegible] where he treats of fluids in gene[illegible] the use of his name and authority, to [illegible] credit and reputation, to some of the [illegible] wild and romantic theories in medi[illegible] and false principles of na[illegible] philosophy, that ever were formed by [illegible] fallible mind; which they indeed de[illegible] from this imaginary source, that has no existence in nature. And, though I [illegible] pretend, in this place, to expose par[illegible]ly such absurdities; yet they ought [illegible] held forth to the world in their true [illegible] genuine deformity, and to be separated [illegible] the name and authority of a man who never gave them the smallest counte[illegible]

PART II.

A NEW METHOD of deriving the THEORY of PROJECTILES in VACUO, from the properties of the SQUARE and RHOMBUS.

PROP. I. THEOR.

IF from one of the angular points of a ſquare, with a diſtance equal to one of its ſides, a circle be deſcribed, and a right line be drawn cutting the circle, and parallel to any one of the ſides; two rhombuſes deſcribed on the ſide to which it is parallel, and having reſpectively one of their angles in the points where it interſects the circle, are ſuch, that their diagonals form equal angles with the diagonals of the ſquare.

For,

For, let ABCD be the ſquare, A the angular point, and DBQ the circle. Let TH be the right line, E, H the points where it interſects the circle, and AETD, AHGD rhombuſes deſcribed on the ſide AD, to which the right line TH is parallel, and having reſpectively one of their angles in the points E, H. Draw the diagonals of the ſquare and rhombuſes, and produce DA till it meet the circle in L.

Then, ſince AB is perpendicular to TM, EI is equal to IH, (3. E. 3.): Alſo, ſince KI is equal to GH, being each equal to AD, (34. E. 1.), and GI common to both, KG is equal to IH, that is, equal to EI. And ſince EG is common to KG, EI; KE is equal to GI, (ax. 3.). In like manner, ſince TE is equal to KI, and KE common to both, TK is equal to EI, that is, to KG. Conſequently, ſince DC is perpendicular to TM, DT is equal to DG, and the angle TDK equal to the angle KDG, (4. E. 1.). Wherefore, a circle deſcribed about the centre D, with the diſtance DA, will paſs through the four

points

points T, C, G, A; and ſince the angles TDC, CDG, have been proved to be equal, their halves TAC, CAG, (20. E. 3.), are likewiſe equal. Alſo, ſince the angles EAB, BAH are equal, (4. E. 1.); their halves EDB, BDH, (20. E. 3.) are alſo equal. Q. E. D.

COROLLARY 1. KE is equal to GI.

Cor. 2. The four right lines TK, KG, EI, IH, are equal; the four angles TDK, KDG, EAI, IAH are equal; and the four triangles TDK, KDG, EAI, IAH are equal in all reſpects.

Cor. 3. The two triangles KDE, GAI are equal in all reſpects.

Cor. 4. The ten angles KDE, GAI, DAF, DTF, FTE, FAE, ADN, NDG, NHG, NHA are equal among themſelves.

Cor. 5. The eight triangles DAF, DTF, FTE, FAE, ADN, NDG, NHG, NHA are every way equal among themſelves.

Cor. 6. The angles GAI, TAI, or the angles EDK, HDK, are together equal to a right angle.

Cor. 7. The angle DAE is equal to twice the angle GAI, and the angle EAL is equal to twice the angle TAI.

As we may eaſily derive from this figure, without drawing any more lines, ſeveral concluſions, which may be uſefully applied to the theory of projectiles *in vacuo*, we have thought proper to inſert ſome of them in the following articles.

Article 1. The ſame things being ſuppoſed, which are mentioned in the propoſition, the ſide of the ſquare, perpendicular to the right line cutting the circle, or any multiple or part of it, has to the perpendicular diſtance of the angular point, about which the circle is deſcribed from the ſaid right line, or to any multiple or part of it reſpectively, the ratio of radius to the ſine of twice either of the angles formed by this ſide, and the diagonals

gonals of the rhombuſes. That is, AB, or any multiple or part of it, has to AI, or any multiple or part of it reſpectively, the ratio of radius to the ſine of twice the angle GAI, or of twice the angle TAI.

For it is evident, that, to the radius EA, which is equal to AB, AI is the ſine of the angle AEI, which is equal to the angle DAE (29. E. 1.), which is equal to twice the angle GAI (Cor. 4.); and that, to the ſame radius, it is likewiſe the ſine of the angle EAL, which is the ſupplement of the angle DAE, and is equal to twice the angle TAI (Cor. 7.); and equimultiples, or like parts of theſe, are proportional to them, (4. and 15. E. 5.) Q. E. D.

Cor. 1. Whence it is plain, that, if either of the angles GAI, TAI, and AI, or any multiple or part of it, be given, AB, or any multiple or part of it, may be found; or, if either of theſe angles and AB, or any multiple or part of it, be given, AI, or any multiple or part of it, may be found; or, if

AB,

AB, AI, or like multiples or parts of them, be given, either of the angles GAI, TAI, may be found.

Cor. 2. Hence it appears, that the perpendicular altitude or breadth of either of theſe rhombuſes, is as the ſine of twice either of the angles which their diagonals form with the ſide of the ſquare.

Cor. 3. Inſtead of the angles GAI, TAI, may be taken the angles TAD, GAD, which are equal to them.

Article 2. Any ſide of the ſquare, or any multiple or part of it, and the ſegments of the right line cutting the circle, intercepted between the diagonals of the rhombuſes, and the ſide of the ſquare, perpendicular to it, or the ſame multiples or parts of them, have to each other reſpectively the ratios of the verſed ſines of twice the angles, which the ſaid ſide of the ſquare forms with the diagonal of the ſquare, and theſe diagonals; that is, the right lines CB, GI, TI, or any like

like multiples or parts of them, have to each other respectively the ratios of the versed sines of twice the angles CAB, GAI, TAI.

For AD, which is equal to BC (34. E. 1.) is evidently the versed sine of the angle DAB, which is equal to twice the angle CAB, to the radius AL, which is equal to AE. It is likewise plain, that KE, which is equal to GI (Cor. 1.), is the versed sine of the angle DTE, which is equal to the angle DAE (34. E. 1.), which is equal to twice the angle GAI, to the radius TE, which is equal to AE. Also it appears, that TI is the versed sine of the angle TEA, which is equal to the angle EAL (29. E. 1.), which is equal to twice the angle TAI, to the same radius EA. And like multiples or parts of these are proportional to them (4. and 15. E. 5.) Q. E. D.

Cor. 1. If any rhombus whatever, as AETD, or ADGH, be described on AD, the perpendicular distance TI or GI from the extremity T, or G, of the diagonal AT, or AG

AG from AB, is always as the verſed ſine of twice the angle TAI or GAI, which the diagonal forms with AB.

Cor. 2. Inſtead of the angles GAI, TAI, may be taken reſpectively the angles TAD, GAD, or the angles EDK, HDK, or the angles HDA, EDA.

Cor. 3. It is alſo plain, that, if CB and either GI or TI be given, the angles GAI, TAI, and TI or GI, may be found; or, if BC and either of the angles GAI, TAI be given, the lines TI, GI, and the angle TAI or GAI, may be found; or, if the angle GAI and GI, or the angle TAI and TI, be given, CB, and the angle TAI and TI, or the angle GAI and GI, may be found.

Article 3. The ſide of the ſquare has to itſelf, and the perpendicular length or breadth of either of the rhombuſes, has to the ſegments of the right line, drawn cutting the circle, intercepted between their diagonals and the ſide of the ſquare perpendicular to it,

it, reſpectively the ratios of radius to the tangents of the angles which the diagonal of the ſquare, and theſe diagonals, form with the ſaid ſide of the ſquare; and the ſame holds of their like multiples or parts.

For AB, or any multiple or part of it, has to CB, or the ſame multiple or part of it, and AI, or any multiple or part of it, has to GI and TI, or the ſame multiples or parts of them, the ratios of radius reſpectively to the tangents of the angles CAB, GAI, TAI. Q. E. D.

From the three articles now demonſtrated, the whole doctrine of projections on a plane paſſing through AB, and having DA, CB, perpendicular to it, may eaſily be derived, as will be ſhown afterwards. And, by applying the demonſtrations given with regard to TI, CB, GI, and the angles TAI, CAI, GAI, to HK, CB, EK, and the angles HDK, BDC, EDK, we make them ſerve for projections on a plane paſſing through DC, parallel to the other.

Article 4.

Article 4. The perpendicular length or breadth of either of theſe rhombuſes, is a mean proportional between the ſegments of the right line which cuts the circle, intercepted between their diagonals, and that ſide of the ſquare which cuts the ſaid line; that is, AI is a mean proportional between TI and GI.

For, ſince the angle GAI is equal to the angle ATI, and the angle AIG is common to both the triangles GAI, TAI, TI has to AI the ratio of AI to IG, (4. E. 6.) Q. E. D.

Cor. AI is a mean proportional between HK and KE.

Article 5. The right line, compounded of the ſegments of the right line cutting the circle, intercepted between the ſide of the ſquare and the diagonals of the rhombuſes, has to theſe ſegments the duplicate ratios alternately of theſe diagonals to that part of the ſaid ſide of the ſquare lying between the centre of the circle, and the right line cutting

ting it. That is, TI, GI together, have to GI the duplicate ratio of AT to AI; and to TI the duplicate ratio of AG to AI.

For, ſince AI is a mean proportional between TI and GI, (art. 4.), the ſquare on AI is equal to the rectangle contained by TI, GI, (17. E. 6.). And, ſince every ſquare and rectangle is a parallelogram, the ſquare on TI is to the rectangle under TI, GI or the ſquare on AI, as TI to GI, (1. E. 6.). Wherefore, the ſquares on TI, AI together, or the ſquare on TA, (47. E. 1.) are to the ſquare on AI, as TI, GI together are to GI, (18. E. 5.). In like manner, it is ſhown, that the ſquares on GI, AI together, or the ſquare on GA, are to the ſquare on AI, as TI, GI together are to TI. Q. E. D.

Cor. 1. The ſame holds with reſpect to HK, KE.

Cor. 2. The ſquare on AI is equal to the rectangle contained under TI, GI, or under HK, KE.

Cor. 3. Inftead of TI, GI together, or HK, KE together, may be taken DL the diameter of the circle.

Cor. 4. The fquare on AT is equal to the rectangle contained by TI or HK, and TI, GI together, or HK, KE together; or by TI or HK, and the diameter DL.

Cor. 5. AT is a mean proportional between DL, and IT, or HK.

Cor. 6. The fquare on AG is equal to the rectangle contained by TI, GI together, or HK, KE together, and GI or KE; or by DL and GI or KE.

Cor. 7. AT is a mean proportional between DL and HK, or the difference of DL and KE.

Cor. 8. AG is a mean proportional between DL and KE, or the difference of DL and HK.

Cor. 9. GI has to TI the duplicate ratio of AG to AT, or the ratio of their ſquares.

Cor. 10. Whence the ſolution of this problem, from either of the acute angles of an iſoſceles right angled triangle, or from the greateſt of them, if it is not iſoſceles, to draw a right line to the oppoſite ſide, which has to the hypothenuſe the ſubduplicate ratio of the ſegment, which it cuts off from that ſide, to the ſide itſelf.

Article 6. The ſemi-diagonals of the rhombuſes are mean proportionals between the radius of the circle, and the halves of the ſegments of the right line, cutting the circle, intercepted between the diagonals, and the ſide of the ſquare; that is, the half of AG is a mean proportional between AE and the half of GI or of EK, and the half of AT between AE and the half of TI, or of HK.

For,

For, by (Cor. 4. and 6. to Art. 5.), the ſquares on theſe diagonals are reſpectively equal to rectangles, contained by twice the radius AE or AD, and the ſegments TI, GI. Therefore the ſquares on their halves, which are the fourth parts of their ſquares (4. E. 2.), are equal to the rectangles contained by the radius and the halves of TI, GI; and conſequently are mean proportionals between the radius and the halves of theſe ſegments (17. E. 6.) Q. E. D.

Cor. 1. FE is a mean proportional between TE and the half of KE; and TF a mean proportional between TE and the half of TI.

Cor. 2. Wherefore, (8. E. 6.) a right line, drawn from F to meet TE, and parallel to DC, will be a mean proportional between the half of KE, and TK with the half of KE taken together.

PROP.

PROP. 2. THEOR.

In BA, CD, two oppoſite ſides of a ſquare ABCD produced, take two equal right lines AP, DM. Join AM, and about the centre P, with the diſtance PD, or AM, deſcribe a circle DQY. From the point Q, draw QR, a tangent to it; and on AM, deſcribe a ſquare AONM. The diagonals AN, AC, of theſe ſquares, will form an angle equal to the angle DAM, and the diagonal AR, of the rectangle AQRD, will biſect the angle NAC, formed by the diagonals of the ſquares.

For, ſince the angle MAN is equal to the angle DAC, the angle MAD, the exceſs of the angle MAC, above the angle DAC, is equal to the exceſs of the angle MAC, above the angle MAN. But this exceſs is the angle NAC. Wherefore, the angle NAC is equal to the angle MAD. Alſo, ſince MR is equal to PQ, or AM, AR is the diagonal of a rhombus deſcribed on AM, and having three of its angular points, in the points A,

A, M, R; and consequently bisects the angle MAB. Therefore, since the angle MAN is equal to the angle CAB, it also bisects the angle NAC. Q. E. D.

Cor. 1. The angles MAD, OAB, NMC, NOC, are equal among themselves, and each equal to NAC.

Cor. 2. The angle NAD is equal to the angle CAO.

If any right line TH be drawn parallel to QR, and cutting the circle in the points E, H, and if ET, HG, be taken each equal to AD, and the rhombuses AETD, ADGH, be completed, several conclusions may be drawn, which may be usefully applied to the theory of projectiles on ascents *in vacuo*, which conclusions are contained in the following articles.

Article 1. The angles, which the diagonals of the rhomboids form with the sides of the square, are correspondently equal, and the

the triangles which they form with the ſaid ſides, and the right line cutting the circle, correſpondently equal in all reſpects; that is, the angles EDK, TAI, are reſpectively equal to the angles GAI, HDK, and the triangles EDK, TAI reſpectively, every way equal to the triangles GAI, HDK.

For EI is equal to IH, (3. E. 3.). Conſequently, ſince KI is equal to GH, KE is equal to GI. But DK is equal to AI, and the angles DKE, AIG are right ones. Therefore, DE is equal to AG, the angle KDE to the angle GAI, and the angle KED to the angle AGI, (4. E. 1.). And, in like manner, it is ſhown, that the angle TAI is equal to the angle HDK, the angle ATI to the angle DHK, and AT to DH. Q. E. D.

COR. 1. KE is equal to GI.

Cor. 2. The right lines TK, KG, EI, IH, are equal among themſelves.

Cor. 3.

Cor. 3. The triangles TDK, KDG, EAI, IAH are equal in all reſpects among themſelves.

Cor. 4. DE is equal to AG, and AT to DH.

Article 2. The angle GAO, TAD, which the diagonals AG, AT, form with the ſides of the ſquares are equal. And the angle TAM is equal to the angle GAB.

For, if a right line were drawn to touch the circle in the point D, it would be perpendicular to a right line drawn from P to D, and conſequently would form, with DK, an angle equal to the angle OAB, and, with DE, an angle equal to the angle GAO. But the angle it would form with DE would be equal to the angle DLE in the alternate ſegment, that is, to the angle DAT, ſince AT is parallel to LE. Therefore, the angle GAO is equal to the angle TAD. Q. E. D.

Cor. 1.

Cor. 1. The angle GAO is equal to each of the angles DAT, ATG, ADH, DHE.

Cor. 2. The angle GAC is equal to the angle TAN, by (cor. 2. to prop. 2.)

Cor. 3. The angle RAG is equal to the angle RAT.

Cor. 4. The triangles GAS, TAS, are ſimilar.

Cor. 5. The triangles TFE, DAF, DAV, HGV are equal and ſimilar, and are ſimilar to the triangles DLE, ATS, SAG.

Cor. 6. The angles AGS, TAS, DAG, ADE, DET, AGH are equal among themſelves.

Cor. 7. Since the angle DAR is equal to the angle RAO, and equal to the angle ARZ (29. E. 1.), AZR is an iſoſceles triangle, and conſequently AD or RQ is equal to AZ, ZQ taken together.

Cor. 8. The angle RAB, which is the half of the angle MAB, is equal to half a right angle together with half the angle OAB, and DAR falls ſhort of half a right angle, by half the ſame angle OAB.

Article 3. The diagonals, AG, AT, of the rhomboids, are reſpectively mean proportionals between DL, or twice DA, and the ſegments GS, TS, of the right line cutting the circle, lying between them and the ſide AO of the ſquare AONM, the ſides of which form equal angles with the ſides of the original one ABCD.

For, ſince the triangles DFA, GSA, TSA, are ſimilar (Cor. 5. to art. 2.), twice AD or DL has to twice DF or AG, the ratio of AG to GS; and twice AD or DL has to twice AF or AT, the ratio of AT to TS. Q. E. D.

Cor. 1. The ſegment GS has to the ſegment TS, the duplicate ratio of the diagonal AG to the diagonal AT.

Cor.

Cor. 2. The ſquares on AG, AT are reſpectively equal to the rectangles contained by DL, GS, and DL, TS; or by DL and the difference between KE, IS, and HK, IS.

Article 4. The diagonals AG, AT have to each other, the ratio of the ſines of the angles GAS, TAS, which they form with AO.

For TS has to GS the ratio of the triangle TAS to the triangle GAS (1. E. 6.), that is, the ratio compounded of the ratios of AT to AG, and of the ſine of the angle TAS to the ſine of the angle GAS. But TS has to GS the duplicate ratio of AT to AG (Cor. 1. to art. 3.). Wherefore twice the ratio of AT to AG is equal to the ratio compounded of the ratios of AT to AG, and of the ſine of the angle TAS to the ſine of the angle GAS. Conſequently the ſines of theſe angles have to each other the ratio of AT to AG. Q. E. D.

Cor.

Cor. Whence, TS has to GS the duplicate ratio of the ſine of the angle TAS, to the ſine of the angle GAS.

Article. 5. DL, or any multiple or part of DL, has to AS, or the like multiple or part of AS, the ratio which is compounded of the ratios, which the ſine of the angle DAS, or TSA, has reſpectively to the ſines of the angles GAS, TAS.

For 2AD or DL has to 2DF or AG, the ratio of AG to GS, that is, of the ſine of the angle ASG or DAS, to the ſine of the angle GAS. And AG has to AS, the ratio of the ſine of the angle ASG or DAS, to the ſine of the angle AGS or TAS. Therefore, the ratio compounded of the ratios of DL to AG, and of AG to AS, that is, the ratio of DL to AS, is equal to the ratio compounded of the ratios, which the ſine of the angle DAS has reſpectively to the ſines of the angles GAS, TAS. Q. E. D.

The

The ſame holds with regard to equal multiples or parts of DL, AS.

Cor. 1. Whence, it appears, that when DL is invariable, AS is as the rectangle contained by the ſines of the angles GAS, TAS directly, and the ſquare on the ſine of the angle DAS inverſely, or the ſquare on the coſine of the angle OAB.

Cor. 2. When the angle OAB is invariable, and conſequently the angle DAS, AS is as the ſolid under the ſines of the angles GAS, TAS, and DL.

Cor. 3. When the angles GAS, TAS are invariable, their ſum is invariable, and the angles DAS, OAB; and then AS is directly as DL.

Cor. 4. When the angle GAS is invariable, AS is directly as the rectangle contained by DL, and the ſine of the angle TAS; and inverſely, as the ſquare on the ſine of the angle DAS,

Cor.

Cor. 5. When the angle TAS is invariable, AS is directly as the rectangle contained by DL, and the ſine of the angle GAS; and inverſely, as the ſquare on the ſine of the angle DAS.

Cor. 6. When either of the angles GAS, TAS and DL are invariable, AS is as the ſine of the other directly, and as the ſquare on the ſine of the angle DAS inverſely.

Cor. 7. When either of the angles GAS, TAS, and the angle DAS or OAB, are invariable, AS is directly as DL.

Cor. 8. When AS is invariable, DL is as the ſquare on the ſine of the angle DAS directly, and the rectangle contained by the ſines of the angles GAS, TAS inverſely.

Cor. 9. When the angle OAB, and conſequently, the angle DAS is invariable, DL is directly as AS, and inverſely, as the rectangle contained by the ſines of the angles GAS, TAS.

Cor. 10. When the angles GAS, TAS, and consequently their sum DAS, is invariable, DL is directly as AS.

Cor. 11. When either of the angles GAS, TAS is invariable, DL is directly as the solid, having the square on the sine of the angle DAS for its base, and AS for its altitude, and inversely, as the sine of the other of these angles.

Cor. 12. When either of the angles GAS, TAS and AS is invariable, DL is directly as the square on the sine of the angle DAS, and inversely as the sine of the other of these angles.

Cor. 13. When either of the angles GAS, TAS and the angle DAS are invariable, DL is directly as AS.

Cor. 14. When DL is invariable, the square on the sine of the angle DAS is directly as the rectangle under the sines of the angles GAS, TAS and inversely as AS.

Cor.

Cor. 15. When AS is invariable, the ſquare on the ſine of the angle DAS is directly as the ſolid under DL, and the ſines of the angles GAS, TAS.

Cor. 16. When DL and AS are invariable, the ſquare on the ſine of the angle DAS, is directly as the rectangle contained by the ſines of the angles GAS, TAS.

Cor. 17. When either of the angles GAS, TAS is invariable, the ſquare on the ſine of the angle DAS is directly as the rectangle contained by DL and the ſine of the other, and inverſely as AS.

Cor. 18. When either of the angles GAS, TAS, and AS are invariable, the ſquare on the ſine of the angle DAS is directly as the rectangle contained by DL and the ſine of the other.

Cor. 19. When either of the angles GAS, TAS, and DL are invariable, the ſquare on the ſine of the angle DAS is directly

rectly as the ſine of the other, and inverſely as AS.

Cor. 20. When DL is invariable, the rectangle contained by the ſines of the angles GAS, TAS are directly as the ſolid, having the ſquare on the ſine of the angle DAS for its baſe, and AS for its altitude.

Cor. 21. When AS is invariable, the rectangle contained by the ſines of the angles GAS, TAS is directly as the ſquare on the ſine of the angle DAS, and inverſely as DL.

Cor. 22. When DL and the angle DAS are invariable, the rectangle contained by the ſines of the angles GAS, TAS is directly as AS.

Cor. 23. When AS and the angle DAS are invariable, the rectangle contained by the ſines of the angles GAS, TAS is inverſely as DL.

Cor. 24. When DL is invariable, the ſine

 of

of either of the angles GAS, TAS is directly as the ſolid, having the ſquare on the ſine of the angle DAS for its baſe, and AS for its altitude, and inverſely as the ſine of the other.

Cor. 25. When AS is invariable, the ſine of either of the angles GAS, TAS is directly as the ſquare on the ſine of the angle DAS, and inverſely as the rectangle contained by DL and the ſine of the other.

Cor. 26. When the angle DAS is invariable, the ſine of either of the angles GAS, TAS is directly as AS, and inverſely as the rectangle contained by DL and the ſine of the other.

Cor. 27. When DL and the angle DAS are invariable, the ſine of either of the angles GAS, TAS is directly as AS, and inverſely as the ſine of the other.

Cor. 28. When AS and the angle DAS are invariable, the ſine of either

of

of the angles GAS, TAS is inversely as the rectangle contained by DL and the sine of the other.

From these articles and their corollaries, the whole doctrine of projections made on ascents *in vacuo*, is easily derived.

PROP. 3. THEOR.

In BA, CD two opposite sides of a square ABCD, take two equal parts AP, DM; join A, M, and about the center P, with the distance PD or AM, describe a circle DQY. From the point Q draw QR a tangent to it, and on AM describe a square AONM. The diagonals AN, AC of these squares will form an angle equal to the angle DAM, and the diagonal AR of the rectangle AQRD will bisect the angle NAC, formed by the diagonals of the squares.

We have the enunciation of this proposition, the same with that of proposition second, excepting only, that there the opposite sides

ſides of the ſquare in which the right lines AP, DM are taken, are produced; whereas here they are not. There the ſegment DQL is leſs than a ſemicircle; here it is greater. There AO forms an angle with AB, on the ſide towards DC; here it forms an angle with it on the other ſide.

This propoſition is demonſtrated in the ſame way with propoſition ſecond; and the ſeveral articles, with their corollaries, deduced from propoſition ſecond, are applicable to this, if the ſame letters be made uſe of in both figures. Only it may not be improper to obſerve, that, whereas, in figure 2d, AZ was equal to the difference of AB and QZ; here it is equal to their ſum; that, whereas there the angle RAQ exceeded 45° by half the angle OAB; here it falls ſhort of 45° by half that angle.

From the articles derived from propoſition ſecond, and their corollaries, with the exceptions already ſpecified, the whole doctrine of projections made on deſcents is eaſily deduced.

PROP.

PROP. IV. THEOR.

If, from one of the angles of a rhombus, a right line be drawn, perpendicular to one of the ſides, containing that angle, to interſect the oppoſite ſide, and from the other extremity of the firſt mentioned ſide a diagonal be drawn; any other rhombus, having one of its angles in the firſt mentioned right line, and the angle oppoſite to this, in that angle of the original rhombus, which is oppoſite to the firſt mentioned angle, will have its angle correſponding to the angle, from which the diagonal is drawn, always further from the firſt mentioned angle than the correſponding point in the ſaid diagonal.

Let AFBD be the rhombus, AB the diagonal, and MN a right line paſſing through the angular point D, perpendicular to AD. Take any part AI of the diagonal, and AH equal to it in the diagonal produced. Through I, H, draw right lines parallel to AD, as QIO, MHK. Then rhombuſes QOFR, MKFP, having reſpectively one of their

angles

angles in the points Q, M, and their angles opposite to these in F, will have their angular points O, K, farther from the angle D, than the corresponding points in the diagonal AB, if right lines be drawn from D to O, K, or will be between the diagonal AB and BF. For, draw the diagonal DF, and the right, lines DI, IF; DH, HF; and AC, KT, OS, perpendicular to BF, produced if necessary.

Then, it is evident, that DI is equal to IF, and DH to HF, (4. E. 1.), and that DI, DH, are respectively greater than QI, MH, (19. E. 1.). Whence, it is plain, that rhombuses having respectively one of their angles in the points Q, M, the angles opposite to these in the point F, and one of their sides respectively coinciding with QI, MH, will have the extremities OK, of these sides farther from the points QM, than the points I, H; and that, consequently, right lines drawn from D to OK, will intersect the diagonal AB. Q. E. D.

Cor.

Cor. 1. Whence it appears, that, if the rhombus AFBD be ſuppoſed to increaſe and decreaſe by motion in ſuch a manner, that AD may always be perpendicular to MN, and two of its oppoſite angles always be in F, and the right line MN, the angular point A will trace out a curve line KAOVG.

Cor. 2. The right line LF is biſected in the point V.

Article 1. The right lines VS, VC, VT, have to each other reſpectively the ratios, which are duplicate to the ratios which the correſponding right lines OS, AC, KT, or QL, DL, ML, have reſpectively to each other.

For OS, AC, KT, are reſpectively mean proportionals between RS, LF; BC, LF; PT, LF, (art. 4. and cor. 1. to prop. 1.); that is, between VS and four times LV, VC and four times LV, VT and four times LV; ſince FS, FC, FT, are equal reſpectively to RL,

RL, BL, PL, (cor. 2. to prop. 1.), and LV equal to VF, (cor. 2. to prop. 4.). Q. E. D.

Cor. 1. VS, VC, VT, are to each other respectively, as the squares on OS, AC, KT, or the squares on QL, DL, ML.

Cor. 2. RV, BV, PV, are equal respectively to VS, VC, VT.

Article 2. If AI be equal to AH, IO is equal to HK.

For DB is a fourth proportional to the sine of the angle DAF of the rhombus, radius, and DL. And ZL, which is a fourth proportional to BC, or the difference of twice BD and LF, AC and BL, or the difference of BD and LF, taken from DL, leaves ZD a fourth proportional to the difference of twice DB and LF, DB and DL; consequently QZ is equal to a right line, which is equal to the difference between fourth proportionals, to the difference of twice BD and LF, BD and DL, LF and DQ, taken together,

gether ; and twice BD, and DQ. And ZM is equal to the difference betwixt the two firſt taken together, and the third of fourth proportionals to the difference of twice BD and LF, BD and DL; twice BD and DQ; LF and DQ. Whence, QI is equal to the difference betwixt the two firſt taken together, and the third of fourth proportionals to DL, DB, and DL; DL, LF and DQ; DL, 2DB and DQ; and MH is equal to the difference between the firſt and third of theſe fourth proportionals taken together, and the ſecond. Conſequently, the difference of MH and QI is equal to the difference betwixt twice a fourth proportional to DL, twice DB and DQ, and twice a fourth proportional to DL, LF, and DQ. But, it is eaſily demonſtrated from (art. 1.) that the difference of MK and QO is equal to twice a fourth proportional to LF, DL, and DQ, which is equal to the ſaid difference expreſſing that of MH, QI; ſince FL is to FE, as DF to DB, a circle paſſing through the four points D, E, L, B. Wherefore, IO is equal to HK. Q. E. D.

Cor. 1. The triangle ELF is iſoſceles, EL being equal to EF.

Cor 2. If from the extremity D, of the baſe DF, of any iſoſceles triangle DBF, a perpendicular DL be drawn to the oppoſite ſide, the ratio of DL to DB is equal to the ratio compounded of the ratios of DL to DF, and of twice LF to DF.

Article 3. AI is a mean proportional between four times AD and IO.

For ſince QO is equal to LV with VS, that is, to the half of LF, with a third proportional to twice LF, and QL, or the difference of DL and DQ; it appears from article ſecond, that IO the difference of QO and QI, is a third proportional to twice LF and DQ. But LV has to LC, or DA, or twice LF has to four times AD the duplicate ratio of AC to AB, (art. 5. prop. 1.), that is, the duplicate ratio of ZQ to ZI, or of DQ to AI. Whence the whole is evident. Q. E. D.

Cor.

Cor. 1. IO is a third proportional to twice LF and DQ.

Cor. 2. The ſquare on AI is equal to four times the rectangle contained by DA, IO.

Article 4. Diſtances BV, IO from points B, I, in the tangent to points V, O, in the curve, in directions perpendicular to MN, have to each other the duplicate ratio of the parts AB, AI of the tangent, which they cut off from it.

For, ſince the ſquare on AB is equal to the rectangle contained by BV, and four times LC (cor. 3. to art. 5. prop. 1.); and ſince the ſquare on AI is equal to the rectangle contained by IO and four times AD, that is, four times LC; BV has to IO the ratio of the ſquare on AB to the ſquare on AI (1. E. 6.), that is, the duplicate ratio of AB to AI (Cor. 1. 19. E. 6). Q. E. D.

Cor.

Cor. BV, IO &c. are reſpectively as ſquares or ſimilar figures on AB, AI, &c.

PROP. V. THEOR.

Let there be any number of magnitudes A, B, C, D, &c. having to each other reſpectively the ratios compounded of the ratios, which a like number of magnitudes L, M, N, O, &c. have reſpectively to each other, and of the ratios, which another like number of magnitudes P, Q, S, T, &c. have reſpectively to each other. Then, if L, M, N, O, &c. be equal among themſelves, A, B, C, D, &c. will have to each other reſpectively, the ratios of P, Q, S, T, &c. to each other; and if P, Q, S, T, &c. be equal among themſelves, A, B, C, D, &c. will have to each other reſpectively, the ratios of L, M, N, O, &c. to each other.

For a ratio of equality added to or taken from any other ratio, neither increaſes nor diminiſhes it.

Cor. Whence, inverſely, if when L, M, N, O, &c. are equal among themſelves, the ratios

ratios of A, B, C, D, &c. to each other be equal to the ratios of P, Q, S, T, &c. to each other; and if, when P, Q, S, T, &c. are equal among themſelves, the ratios of A, B, C, D, &c. to each other be equal to the ratios of L, M, N, O, &c. to each other: when the magnitudes in neither of theſe rows are equal among themſelves, A, B, C, D, &c. have to each other reſpectively the ratios compounded of the ratios of both.

PROP. VI. THEOR.

The ſpaces which bodies move through with uniform or invariable velocities, have to each other reſpectively, the ratios which are compounded of the ratios which theſe velocities have reſpectively to each other, and of the ratios, which the times in which they are deſcribed, have to each other reſpectively.

For theſe ſpaces are as the velocities, when the times are equal, and as the times, when the velocities are equal. Therefore,

when

when neither of them are equal, they are as both (Cor. to prop. 5.). Q. E. D.

Cor. When the times are equal, the ſpaces are as the velocities; and, when the velocities are equal, as the times.

PROP. VII. THEOR.

If a body, from a ſtate of reſt, move with a velocity, which receives equal degrees of increaſe in equal portions of time, the ſpaces, which it moves through, have to each other the duplicate ratios of the times, in which it moves through them.

Let ED repreſent the velocity at the end of the time repreſented by AD. Produce AD, AE, and parallel to ED, draw right lines HF, CB, meeting AD, AE produced. Then, ſince the velocities are to each other as the times in which they are produced, HF, BC, will repreſent the velocities at the ends of times repreſented by AF, AB. Whence, if a right line begin to move from the vertex

tex A, of the triangle BAC, parallel always to the baſe BC, the parts of it intercepted between AC, AB, will repreſent the velocities that anſwer to all the points of time which are repreſented by the points in AB. And, it is evident, that theſe velocities, at the ends of the times AD, AF, AB, trace out the ſpaces ADE, AFH, ABC. But (19. E. 6.) theſe ſpaces have to each other reſpectively ratios, which are duplicate to the ratios which DE, FH, BC; AD, AF, AB, have to each other reſpectively. Q. E. D.

Cor. 1. The ſpaces are to each other in the duplicate ratios of the velocities.

Cor. 2. The ſpaces are to each other in the ratios compounded of the ratios of the times and velocities.

Cor. 3. If AD, DF, FB, &c. be equal, ADE, EDFH, HFBC, &c. are as 1. 3. 5. &c.

Cor. 4.

Cor. 4. In the time AB, with the velocity CB, it would deſcribe twice the ſpace BAC.

Cor. 5. The ſame velocity BC, decreaſing in the ſame manner as the velocity was increaſed, vaniſhes in the time AB.

PROP. VIII. THEOR.

A body impreſſed with two uniform velocities in different directions, or forced to move from a ſtate of reſt, with velocities in different directions, which increaſe or decreaſe reſpectively, as the time increaſes or decreaſes, moves in a ſtraight line.

For, let ABED be any parallelogram, AE its diagonal, and AFGC any other parallelogram having its diagonal AG, in the diagonal AE. Then,

Firſt, Let the body be impelled from the point A, with velocities in the directions AB AD, which would carry it through theſe ſeparately

paratelỳ in the ſame time. At the end of this time, then, it will be found in neither of theſe, but at diſtances from AB, AD, in directions parallel to them, reſpectively equal to AD, AB, that is, in the point E. Now, if AF be to AB, as any other portion of time to this time, ſince AC has to AD, the ratio of AF to AB, it will move in the direction AD through AC, in that other portion of time; and therefore, at the end of it, will be in the point G. And as G may be any point in AE, it is plain, that the body moves in the right line AE from A to E. Q. E. D.

Secondly, Let the body at reſt in A, begin to move under the influence of forces, which communicate velocities to it, increaſing as the time increaſes, and which would carry it ſeparately through the diſtances AB, AD in the ſame time. Then at the end of this time it will be in the point E. And, if AF have to AB the duplicate ratio of any other portion of time to this time, it will be in G at the end of that time; ſince, in that time, it would move through AC in the di-

rection AD. And since G may be any point in AE, it is evident, that it will move in the right line AE from A to E.

The same holds when the velocities decrease.

PROP. IX. THEOR.

If a body begin to move in any direction with an uniform velocity, and begin to be acted upon at the same time in another direction, by a force which communicates to it a velocity, which increases as the time increases, it will move in a curve line. See figure 4.

Let a body be projected or made to move in the direction AB with an uniform or invariable velocity, and begin to be acted upon at the same time, in a direction parallel to BF, by a force which communicates to it a velocity increasing as the time increases.

Then it appears from (prop. 7.) that at the ends

ends of the times, in which it would have moved with the uniform velocity from A to I and B, it would have moved in a direction parallel to BF, to points at distances from AB, having to each other the duplicate ratio of AI to AB, that is, the ratio of IO to BV (art. 4. to prop. 4.)

Whence it is evident, that it will move in a curve of the same nature with the curve AOVG.

Cor. Whence all the properties of the curve AOVG, in figure to proposition 4th, may be applied to the curve which a body describes, moving according to the conditions mentioned in this proposition.

SCHOLIUM.

Whence, it is evident, that if, in figure seventh, IO be taken equal to AI, BP to AB, and TI, GI, CB, be respectively bisected in the points V, Y, Z, a body projected with the velocity, which

it

it would acquire in moving from D to A, under the influence of such a force as is mentioned in prop. 7. in the directions AT, AG, AC, would move respectively in the curves AVO, AYO, AZP, which are of the same nature with the curve in the figure to prop. 4. For, with this velocity, it would move over twice AD, in the direction AT. And (cor. 3. to art. 5. prop. 1.), TV has to DA, the duplicate ratio of AT to twice AD.

If then, in compliance with custom, we call AO, AP, the ranges or amplitudes, AD the impetus, AT, AC, AG, the lines of projection, which are always tangents to the curves in the point A, as is evident from prop. 4. VI, ZB, YI, the altitudes of the projections, TAI, CAB, GAI, the angles of elevation, KV, CZ, KY, the sublimities; we have, from article first to proposition first, the following analogy.

1. ANA-

1. ANALOGY.

Radius has to the ſine of twice the angle TAI, or of twice the angle GAI, the ratio of the amplitude AP, to the amplitude AO.

Whence, it appears, that AP is the greateſt poſſible amplitude with the impetus AD.

And, from article 2. to propoſition firſt, we have the following analogy.

2. ANALOGY.

Radius has to the verſed ſine of twice the angle GAI, the ratio of AD, or the half of AP, or twice ZB, to GI or twice YI, that is, of half the impetus, or of the altitude ZB, to the altitude YI.

And radius has to the verſed ſine of twice the angle of elevation TAI, the ratio of the altitude ZB, or half the impetus to the altitude VI.

And

And from article 3. we have the following analogy.

3. ANALOGY.

Radius has to the tangent of the angle of elevation GAI, the ratio of AI, or half the amplitude AO, to twice the altitude YI, to the tangent of the angle of elevation TAI, the ratio of AI or half the amplitude AO, to twice the altitude VI, and to the tangent of the angle of elevation CAB, the ratio of AB, or half the amplitude AP, to twice the altitude ZB.

Or, univerſally, Radius has to the tangent of the angle of elevation, the ratio of one fourth part of the amplitude to the altitude.

The other articles, with their corollaries derived from propoſition firſt, may likewiſe be uſefully applied to particular caſes of ſuch projections.

And,

And, by proceeding in like manner with the figures to propositions second and third, we not only derive from the articles drawn from these propositions and their corollaries, the rules and analogies usually given for projections made on planes inclined to the plane to which the impetus is perpendicular; but likewise a great number of other conclusions, which may be used with advantage on particular occasions.

Thus, all the cases of projections on inclined planes may be solved by means of the three following rules derived from art. 5. and cor. 22. to art. 5. prop. 2. and 3.

1. Having the angle of obliquity, the angle of elevation above the inclined plane, and the range on it given, to find the impetus.

RULE from Art. 5. Prop. 2. and 3.

To the logarithm of one fourth part of the range, add twice the logarithmic sine of the

the angle contained by it and the impetus. Then, from their ſum, ſubſtract the logarithmic ſine of the exceſs of this angle above the angle of elevation, together with the logarithmic ſine of the ſaid angle of elevation, and the remainder will be the logarithm of the impetus.

2. Having the impetus and the angle of elevation above the inclined plane given, to find the range on it.

RULE from the ſame.

To the logarithm of the impetus, add the logarithmic ſine of the exceſs of the angle contained by it, and the range on the inclined plane above the angle of elevation, together with the logarithmic ſine of the ſaid angle of elevation. Then, from their ſum, ſubtract twice the logarithmic ſine of the angle contained by the impetus and range, and the remainder will be the logarithm of one fourth part of the range on the inclined plane.

3. Having

3. Having one angle of elevation above the inclined plane, and the range correſponding to it, and another angle of elevation above the plane given, to find the range correſponding to this angle.

RULE, from Cor. 22. to Art. 5. of the ſame.

To the logarithm of the given range, add the logarithmic ſine of the exceſs of the angle contained by it, and the impetus, above the angle of elevation anſwering to the required range, together with the logarithmic ſine of the ſaid angle of elevation. Then, from their ſum, ſubtract the exceſs of that angle, above the angle of elevation, anſwering to the given range, together with the logarithmic ſine of this angle of elevation, and the remainder will be the logarithm of the required range.

The varieties in theſe three rules comprehend all the caſes of projections on inclined planes.

However, as the calculations for ſuch projections are generally reckoned by artilleriſts a good deal more puzzling and perplexing than thoſe made for projections on a plane, to which the impetus is perpendicular, I ſhall ſubjoin ſome rules for reducing the former of theſe to the latter, with the greateſt eaſe and expedition.

RULE I.

When the object or mark to be hit, is elevated above the plane to which the impetus is perpendicular at the point of projection; that is, above the horizontal plane, if we ſuppoſe the impetus to be perpendicular to the horizon;

From the ſquare of the difference betwixt twice the impetus, and the altitude of the elevated object, take the ſquare of the inclined plane in their arithmetical values. Take the ſquare-root of this difference from, and add it to the firſt difference; multiply this difference or ſum by the numerical value

lue of the ſquare of the horizontal diſtance, and divide the product by four times the numerical value of the ſquare of the inclined plane. Multiply the quotient by four times the exceſs of the numerical value of the impetus above it. Take the ſquare root of this product, and the impetus will have to it the ratio of radius to the ſine of twice the angles of elevation; the higheſt of which correſponds to a curve paſſing through the top of the object, and the loweſt to one paſſing through a point in the object between its top and the horizon.

This rule is eaſily applied by means of logarithms.

GEOMETRICALLY.

Let I repreſent the impetus, A the altitude of the mark or object above the horizon, D the horizontal diſtance, and P the inclined plane. On GH equal to the difference of twice I and A, deſcribe a ſemicircle GKH, in which from G one extremity of GH,

draw

draw a right line GK equal to P. Let L be a third proportional to twice P and D. Take GM a fourth proportional to twice P, L and the difference of GH, HK; and GN a fourth proportional to twice P, L and GH, HK taken as one line. Then take Q a mean proportional between GM and four times the difference of I and GM; or between GN and four times the difference of I and GN. With I or OZ describe a quadrant ZSE; at one extremity R of Q draw RS parallel to I or OZ to meet the quadrant in S; draw OS, and bisect the angle ZOS by the right line OC. A body projected in the direction OC with the impetus OZ or I, will describe a curve passing through the top of A. And if RT be equal to twice GM, or the difference of OZ and RS, a body projected in the direction OT with the same impetus, will meet A in a point between its top and the horizon.

RULE. 2.

When the object or mark to be hit, is depressed

preſſed below the horizontal plane, if the impetus be perpendicular to the horizon;

Take from the ſquare of the ſum of twice the impetus, and the perpendicular diſtance of the top of the depreſſed object from the horizontal plane, the ſquare of the inclined plane in their arithmetical values. Take the ſquare root of this differenec from, and add it to the ſaid ſum. Multiply this ſum or difference by the numerical value of the ſquare of the horizontal diſtance, and divide the product by four times the numerical value of the ſquare of the inclined plane. Multiply the quotient by four times the exceſs of the numerical value of the impetus above it. Take the ſquare root of this product, and the impetus will have to it the ratio of radius to the ſine of twice the angles of elevation; the loweſt of which correſponds to a curve paſſing through the top of the object, and the higheſt to one, paſſing through a point farther than that from the horizontal plane.

GEOME-

GEOMETRICALLY.

Let I reprefent the impetus, A the perpendicular diftance of the object from the horizontal plane, D the horizontal diftance, and P the inclined plane. On GH, equal to twice I and A together, defcribe a femicircle GHK, in which from G one extremity of GH, draw a right line GK equal to P. Let L be a third proportional to twice P and D. Take GM a fourth proportional to twice P, L and the difference of GH, HK; and GN a fourth proportional to twice P, L and the fum of GH, HK. Then, take Q a mean proportional between GM, and four times the difference of I and GM; or between GN and four times the difference of I and GN. With I or OZ, defcribe a quadrant ZSE; at one extremity R of Q, draw RS parallel to OR; draw OS, and bifect the angle ZOS by OC, which will be the direction for a body to be projected in, to pafs through a point farther from the horizon than the extremity of A. And, if TR be equal to twice GM, or the difference

of

of OZ and SR, AT will be the di-ection for a body to be projected in, to paſs through the ſame point of the horizon with the former, and through the extremity of A.

RULE 3.

When the horizontal diſtance, the altitude or depreſſion of the object, and either angle of elevation above the horizon are given, and it is required to find the impetus;

Take a fourth proportional to radius, the tangent of the angle of elevation, and the horizontal diſtance. Divide the ſum of the ſquares on this and the horizontal diſtance, in their numerical values, by four times the difference between it and the elevation of the object, or by four times the ſum of it and the depreſſion of the object, in their numerical values, and the quotient will be the numerical value of the impetus.

The geometrical conſtruction is very ſimple.

RULE

RULE 4.

When the impetus, the altitude or depreſſion of the object, and either angle of elevation above the horizon are given, to find the horizontal diſtance, and the range on the inclined plane.

Find the ſublimity correſponding to the angle of elevation, and a fourth proportional to the ſquare on radius, the ſquare on the ſine of this angle, and the impetus. Then find mean proportionals between four times the ſublimity, the difference of this fourth proportional, and the altitude of the object, and the ſum of this fourth proportional and the depreſſion of the object. The firſt of theſe mean proportionals added to twice a fourth proportional to radius, the ſine of this angle and a mean proportional between the impetus and ſublimity, gives the horizontal diſtance in the caſe of aſcents; and the laſt of theſe mean proportionals added to the ſame, gives it in the caſe of deſcents. And the range on the inclined plane is obtained by (47. E. 1.).

The

The geometrical conſtruction here is likewiſe very ſimple.

And, in a ſimilar manner, may all the poſſible variety of caſes for projections on inclined planes be ſolved.

TABLE. I.

For Horizontal Projections.

CASE. 1.

The impetus and amplitude being given, to find the angles of elevation, and the altitudes.

ANALOGIES.

Twice the impetus is to the amplitude, as radius to the ſine of twice either angle of elevation. And radius is to the tangents reſpectively of the angles of elevation, as one fourth part of the amplitude is to the altitudes reſpectively.

CASE. 2.

The impetus and either angle of elevation being given, to find the amplitude.

ANALOGY.

Radius is to the ſine of twice either angle of elevation, as twice the impetus is to the amplitude.

CASE. 3.

The amplitude and either angle of elevation being given, to find the impetus.

ANALOGY.

As the ſine of twice either angle of elevation is to radius, ſo is one half of the amplitude to the impetus.

CASE. 4.

The impetus and either altitude being given, to find the amplitude.

RULE.

RULE.

Take a mean proportional between either altitude, and the exceſs of the impetus above it, and four times this will be equal to the amplitude.

Or thus by LOGARITHMS.

To half the logarithm of four times the altitude, add half the logarithm of four times the exceſs of the impetus above the altitude, and the ſum will be the logarithm of the amplitude.

CASE 5.

The amplitude and altitudes being given, to find the angles of elevation, and the impetus.

ANALOGY for the ANGLES of ELEVATION.

As one fourth part of the amplitude is to the

the altitudes reſpectively, ſo is radius to the tangents of the angles of elevation reſpectively.

RULE for the IMPETUS.

To either altitude, add a third proportional to it, and one fourth part of the amplitude, for the impetus.

CASE 6.

The altitudes and angles of elevation being given, to find the amplitude.

ANALOGY.

The tangents of the angles of elevation are reſpectively to radius, as the altitudes are reſpectively to one fourth part of the amplitude.

CASE 7.

The angles of elevation, the amplitude, and

and any other angle of elevation being given, to find the amplitude correſponding to that other angle.

ANALOGY.

The ſine of twice either angle of elevation is to the ſine of twice that other angle, as the given amplitude is to the amplitude required.

CASE 8.

The angles of elevation, the amplitude, and any other amplitude being given, to find the angles of elevation correſponding to that other amplitude.

ANALOGY.

The amplitude correſponding to the given angles of elevation, is to the other amplitude, as the ſine of twice either of theſe angles, is to the ſine of twice either angle required.

CASE 9.

An angle of elevation, the altitude correſponding thereto, and any other angle of elevation being given, to find the altitude correſponding to that other angle.

ANALOGY.

The verſed ſine of twice the angle of elevation, anſwering to the given altitude, is to the verſed ſine of twice that other angle, as the given altitude is to the altitude required.

CASE 10.

An angle of elevation, the altitude correſponding thereto, and any other altitude being given, to find the angle of elevation correſponding to that other altitude.

ANALOGY.

The altitude correſponding to the given angle

angle of elevation, is to that other altitude, as the verſed ſine of twice this angle is to the verſed ſine of twice the angle required.

TABLE II.

For Projections on Inclined Planes.

CASE I.

The impetus and any two of theſe three things being given, to wit, the horizontal diſtance, the altitude or depreſſion of the object, and the length of the inclined plane, to find the angles of elevation.

RULE.

As the horizontal diſtance is to the altitude or depreſſion, ſo is radius to the tangent of the angle of inclination.

Then, as the tangent of an angle equal to half this angle, together with half a right angle,

angle, is to radius, ſo is half the impetus to a fourth proportional.

Laſtly, As a fourth proportional to the horizontal diſtance, the length of the inclined plane, and half the impetus, is to the exceſs of the fourth proportional, found by the ſecond analogy above one fourth part of the horizontal diſtance, ſo is radius to the verſed ſine of an angle, which added to, and taken from the firſt mentioned angle in the ſame analogy, gives the angles of elevation required.

CASE 2.

The angles of elevation and the length of the inclined plane or amplitude being given, to find the impetus.

ANALOGY.

As the rectangle contained by the ſine of either angle of elevation, and the ſine of the exceſs of the angle formed by the impetus and amplitude above this angle, is to the

ſquare

ſquare on the ſine of the angle formed by the impetus and amplitude, ſo is one fourth part of the amplitude to the impetus.

By LOGARITHMS.

Take the firſt of the three rules immediately preceeding the rules for reducing inclined projections to horizontal ones.

CASE 3.

The angles of elevation and the impetus being given, to find the amplitude.

ANALOGY.

The ſquare on the ſine of the angle formed by the impetus and amplitude, is to the rectangle contained by the ſine of either angle of elevation, and the ſine of the exceſs of the angle formed by the impetus and amplitude above this angle, as the impetus is to one fourth part of the amplitude.

By LOGARITHMS.

Take the ſecond of thoſe three rules.

CASE 4.

An angle of elevation, the amplitude correſponding to it, and any other angle of elevation being given, to find the amplitude correſponding to that other angle.

ANALOGY.

The rectangle contained by the ſine of the angle correſponding to the given amplitude, and the ſine of the exceſs of the angle formed by the impetus and amplitude above this angle, is to the rectangle contained by the ſine of the angle correſponding to the required impetus, and the ſine of the exceſs of the angle formed by the impetus and amplitude above this angle, as the given amplitude is to the required one.

By

By LOGARITHMS.

Take the laſt of thoſe three rules

CASE 5.

The impetus and angle of inclination being given, to find the greateſt poſſible amplitude.

ANALOGY.

The tangent of an angle equal to half a right angle, together with half the angle of inclination, is to the ſecant of the angle of inclination, as half the impetus is to one fourth part of the greateſt amplitude.

RULE for determining the Duration of any Projection.

The coſine of the elevation of the plane above, or depreſſion below the horizon, is to the ſine of the angle of elevation above the plane;

plane, as twice the time of defcent through the impetus is to the time of the flight.

Note. Near the furface of the earth, bodies would defcend in vacuo through about 16 feet 1 inch in a fecond.

Thus have I endeavoured to deduce the theory of projectiles in vacuo, both on horizontal and inclined planes, from the properties of the fquare and rhombus, and by means of fome very eafy and fimple rules, to extend the analogies, given for projections made on the horizon, to thofe which are made on planes inclined to it, in any angles. In every cafe, however, it muft be obferved, that the projectile is fuppofed to be acted upon, in a direction perpendicular to the horizontal plane, by fuch a force as is mentioned in propofition feventh, that is, by a force which acts on it uniformly, and thereby has an uniform influence in accelerating its motion towards the horizon, when it proceeds from a ftate of reft, and in retarding its motion from the fame, when it is pro-

jected

jected with any degree of velocity in ſuch a direction. The ingenious Galileo, who has the honour of having firſt conſidered equably accelerated motion with any degree of accuracy or ſucceſs, has, in the third of his incomparable dialogues, diſcourſed on this ſubject at great length, and ſhown, in the moſt ſatisfactory and unexceptionable manner, that the ſpaces, which a body proceeding from reſt, under the influence or continued action of any uniform force, moves through in different times, have to each other the duplicate ratios of theſe times reſpectively. And, in his fourth dialogue, he demonſtrates, that ſuch an accelerated motion, combined with an uniform one, would make a projectile in vacuo, trace out that conic ſection, which is called the parabola. Beſides, as this great and ſagacious man, has, in the former of theſe dialogues, demonſtrated from experiments, which he repeatedly made, with a ball of the hardeſt braſs finely poliſhed, in a grove about twelve yards long, cut in a priſm of wood, and covered on the inſide with very ſmoothly poliſhed vellum,

lum, that the ſpaces which a body moving from reſt deſcends through, have to each other reſpectively, the duplicate ratios of the times of deſcent; ſo, in the latter of theſe, he concludes, that, on the ſuppoſition of a vacuum or empty ſpace near the ſurface of the earth, the path of a projectile would be nearly in a parabola. I ſay nearly, becauſe, at the ſame time in which he draws this concluſion, he appears to be fully ſenſible, that it is not ſtrictly and accurately true. But, ſince this error is ſo very ſmall if the motion be conceived to be made in vacuo, that the difference betwixt theory and experience, would, on this ſuppoſition, be almoſt altogether imperceptible, he choſe, for the ſake of eaſe and ſimplicity, to omit the conſideration of thoſe circumſtances, which would have only occaſioned ſuch minute variations, but which, if taken into the account, would have rendered his calculations much more complex and intricate. This difference, that would take place betwixt the parabolic theory and experience *in vacuo*,

vacuo, ariſes from two cauſes which I ſhall now endeavour to explain.

In order, then, that a body ſhould move in a parabola under the combined influences of a projectile, and uniformly accelerating force, it is neceſſary, that the direction of this acceleration ſhould be always perpendicular to the ſenſible horizon, at the point of projection, or to a plane touching the earth's ſurface, at that point. This, I ſay, is abſolutely neceſſary, ſince all the diameters of a parabola are parallel to one another. But this is not in reality the caſe with a body thrown from a point, on the earth's ſurface, in any direction. For, that accelerating force which is conſtantly employed in carrying it from the line of projection, towards the ſuperficies, tends always to the center of the earth. Whence, it evidently appears, that thoſe lines, according to which the accelerating force would act in different points of the curve, which the body would really deſcribe, would not be parallel to each other, but would meet at the center of the earth; and

and there form angles with one another.

However, as the greateſt range or amplitude of the largeſt piece of ordinance, on the horizontal plane, were it fired with its uſual allotment of powder, ſuppoſing this to be poſſible *in vacuo*, would bear but a very ſmall proportion to the ſemidiameter of the earth; even the greateſt of theſe angles would be but very inconſiderable; and, therefore, the poſitions of the right lines, extending from different points in the curve to the center of the earth, would differ little from a ſtate of parallelism. If the velocity, indeed, with which a body is ſuppoſed to be projected, be imagined to be very great, the deviation of the curve from the parabolic form would, even *in vacuo*, be very conſiderable. But the circumſtance I have been juſt ſpeaking of is not the only cauſe of this variation. There is another, which I ſhall now mention.

Were gravity to accelerate a body uniformly towards the center of the earth, its force would continue uniform, or the ſame at different diſtances. This, however, we

know

to be ſo far from being the caſe, that its force decreaſes as the ſquares of the diſtances of the body from this center increaſe; that is, at twice the diſtance, it is four times as ſmall, at thrice the diſtance, nine times as ſmall, and ſo on. Now, it is certain, that the point on the ſurface of any ſphere, where a plane touches it, is nearer the center of that ſphere, than any other point in the ſaid plane; and, that conſequently, any line of projection drawn from that point, ſo as to form any angle whatever, with a right line drawn in that plane from the point of contact, will have all its points at different diſtances from the ſame center. Hence it is abundantly obvious, that a projectile would be at different diſtances, during the time of its flight, from the center of a ſphere. And this concluſion evidently holds in greater force with reſpect to the earth, which is not a perfect ſphere, but is of a ſpheroidal figure. This circumſtance then plainly ſhows, that gravity would not accelerate a body uniformly even in vacuo, towards the center of the earth, and that a pro-

jectile would receive different degrees of acceleration at different points in the curve which it would deſcribe. And this deviation from the law of uniform or equable acceleration, would likewiſe vary the concluſion derived from it, which was, that the ſpaces deſcended through were in the duplicate proportion of the times of deſcent. But all the artifices that we could have recourſe to, in trying experiments with regard to falling bodies, would ſcarce render this variation perceptible, ſince, at ſmall diſtances from the earth's ſurface, it is ſo minute as to elude experiment. Whence it is not at all ſurprizing, that Galileo did not perceive it in thoſe experiments, which he made with a ball of braſs, in the groove of a priſm of wood, and elevated, as he himſelf informs us in his third dialogue, one or two yards above the horizon. For, the force of gravity at the elevated end of the priſm, would have to its force at the ſurface of the earth, the duplicate ratio of the ſemidiameter of the earth to a right line

equal

equal to this ſemidiameter with one or two yards, that is, nearly a ratio of equality.

From theſe obſervations, it evidently appears, that, were it not for the reſiſtance of the air, a projectile thrown from any point on the earth's ſurface, would nearly deſcribe the curve generated in the figure to Propoſition fourth, which curve is indeed nothing but the parabola derived after a new manner. But this reſiſtance of the medium produces much more powerful effects on projectiles than both the above mentioned cauſes taken together, and poſſeſſes ſuch a prodigious influence in varying and retarding their motions, as muſt ſeem altogether amazing, when mentioned to any perſon, who is not accuſtomed to conſider ſuch matters with ſufficient care and attention.

FINIS.

Fig. 1.

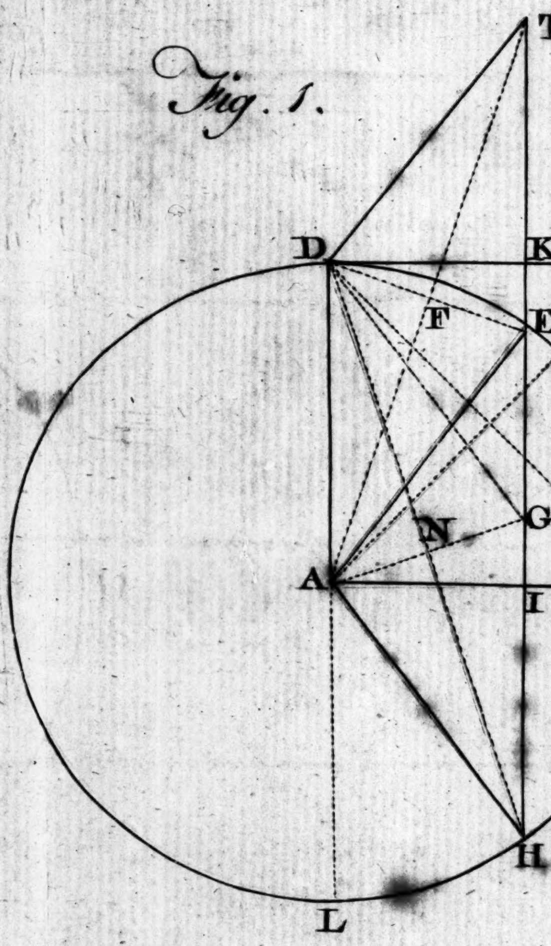


Fig. 3.

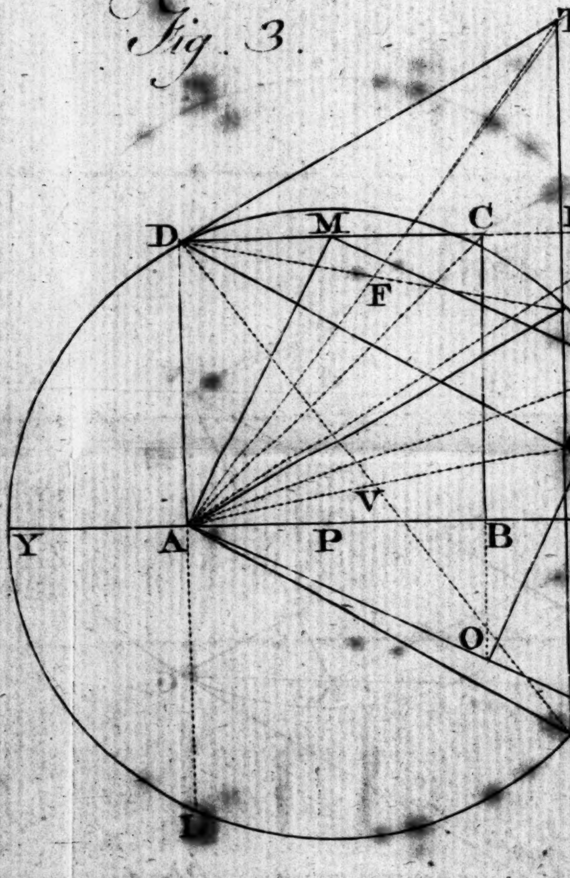


Plate I.

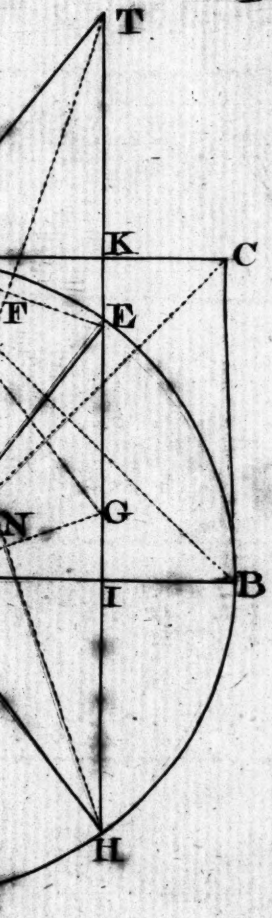
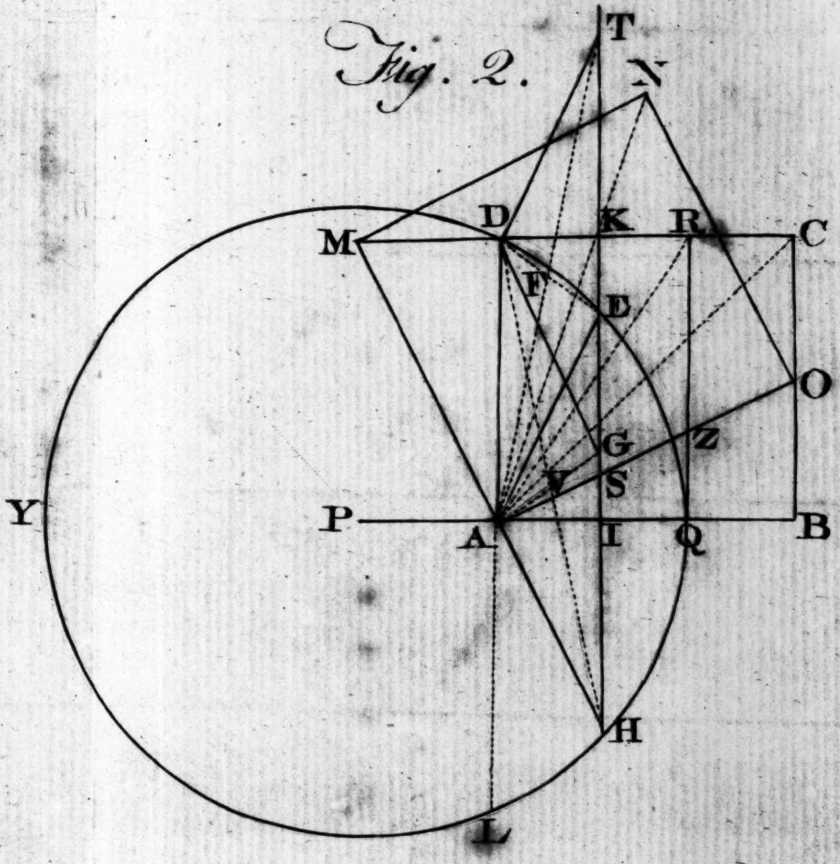
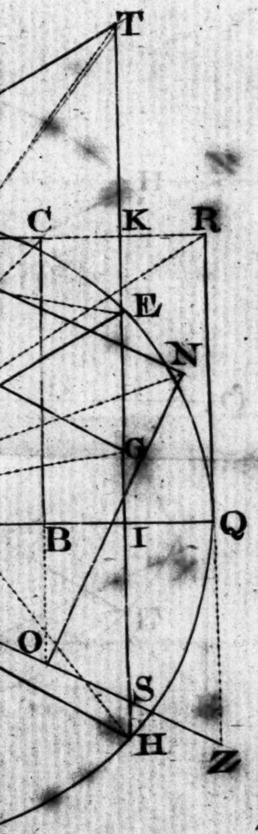


Fig. 2.





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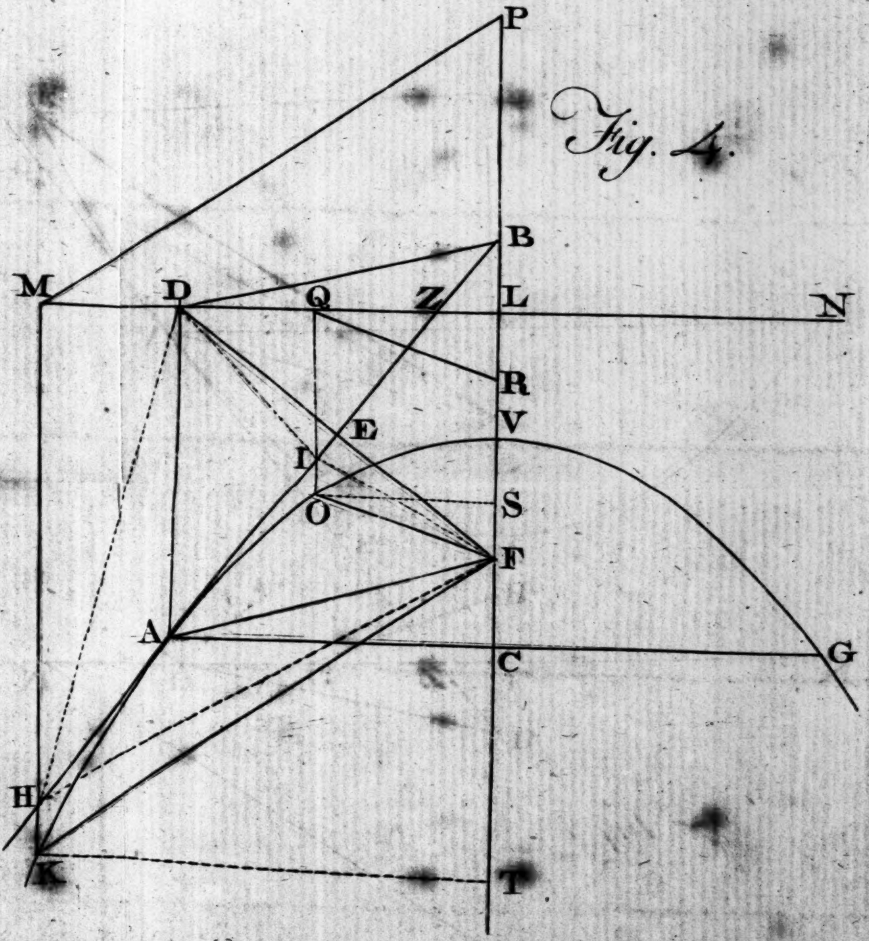

Plate. II.

Fig. 5.

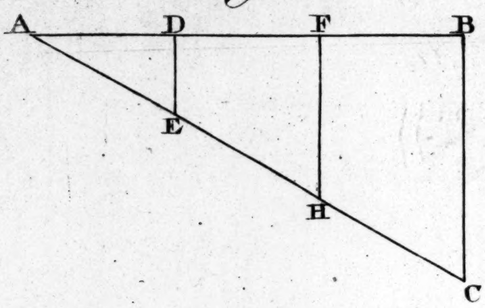

Fig. 6.

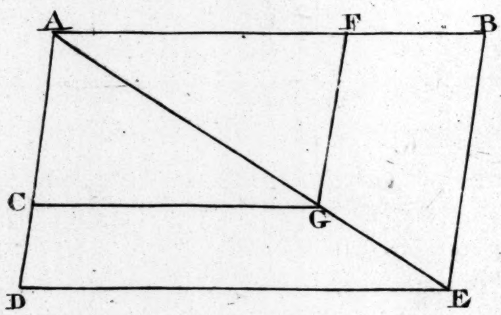

Fig. 7.

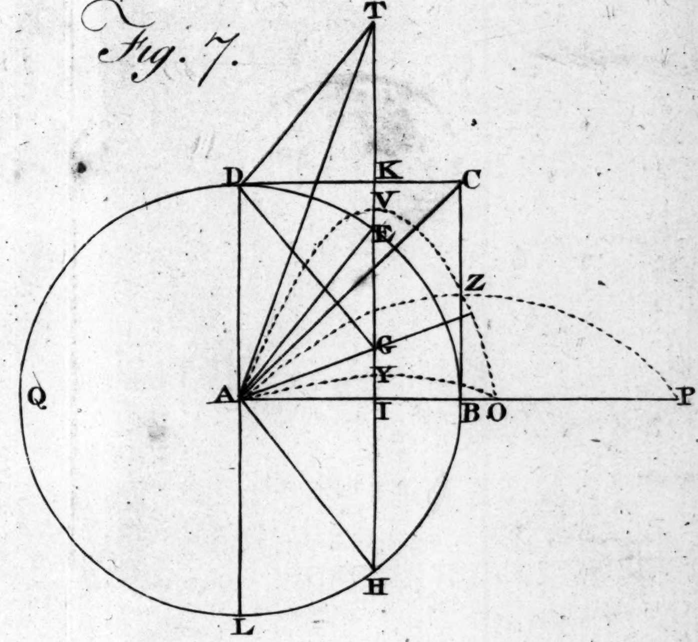

Fig 8

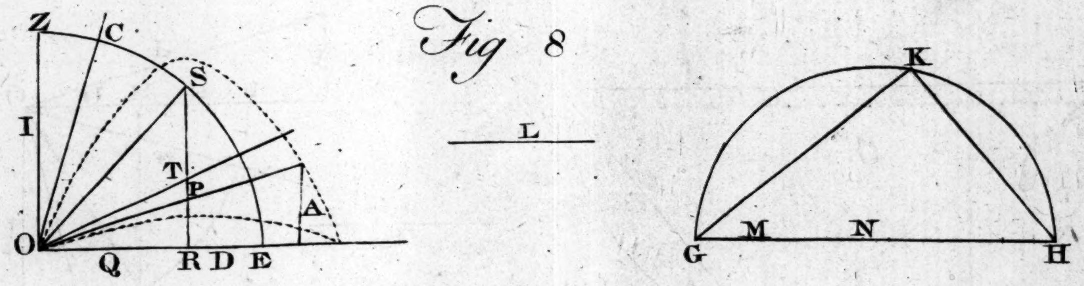

Fig. 9.